Math 257/316 Assignment 7, Due Wednesday Nov. 1 th in class

Note: The final Exam is on Thursday December 7 at 8:30.

**Problem 1:** (Old Exam Question) Solve the initial boundary value problem:

\[
\begin{align*}
  u_t &= u_{xx} + e^{-t}(1 - x), \quad 0 < x < 1, \ t > 0 \\
  \text{BC:} \ u(0, t) &= 1, \ u(1, t) = e^{-t} \\
  \text{IC:} \ u(x, 0) &= 1
\end{align*}
\]

**Problem 2:** Solve the inhomogeneous heat conduction problem subject to time dependent boundary conditions:

\[
\begin{align*}
  u_t &= \alpha^2 u_{xx} - 1 - xe^{-t}, \ 0 < x < 1, \ t > 0 \\
  u_x(0, t) &= e^{-t}, \ \text{and} \ u(1, t) = t \\
  u(x, 0) &= x.
\end{align*}
\]

**Problem 3 (Hand In):** Solve the inhomogeneous heat conduction problem subject to time dependent boundary conditions:

\[
\begin{align*}
  u_t &= u_{xx} + xt + 1, \ 0 < x < 1, \ t > 0 \\
  u_x(0, t) &= 0, \ \text{and} \ u(1, t) = t \\
  u(x, 0) &= 0.
\end{align*}
\]

**EXCEL EXERCISE:** The flux boundary condition at the left of the bar \( x = 0 = x_0 \), can be approximated by the following difference quotient:

\[
\frac{\partial u(0, t)}{\partial x} = \frac{u(x_0 + \Delta x, t) - u(x_0 - \Delta x, t)}{2\Delta x} = 0
\]

Implement this boundary condition in the spreadsheet: **Heat0.xls** posted on the web site by inserting a new column to the left of column B and inserting these fictitious values in the new column B. Plot \( u(x, t = 0.5) \) obtained using the numerical solution, print it out, and hand it in with your assignment.

**Problem 4 (Hand In):** Solve the inhomogeneous heat conduction problem with heat loss, a time dependent source, and subject to time dependent boundary conditions:

\[
\begin{align*}
  u_t &= u_{xx} - u + e^{-t} \sin(x), \ 0 < x < \frac{\pi}{2}, \ t > 0 \\
  u(0, t) &= 0, \ \text{and} \ \frac{\partial u(\pi/2, t)}{\partial x} = e^{-t} \\
  u(x, 0) &= x.
\end{align*}
\]