

INTEGRATING FUNCTIONS ON INFINITE INTERVALS

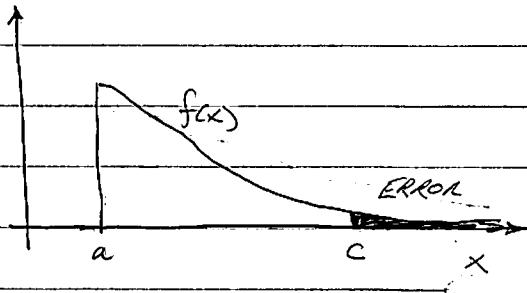
$$I = \int_a^\infty f(x) dx$$

ASSUME $f(x) \sim x^{-p}$ $p > 1$

FOR THE INTEGRAL TO BE DEFINED

1. TRUNCATE THE INFINITE INTERVAL

$$I = \int_0^c f(x) dx + \underbrace{\int_c^\infty f(x) dx}_{\text{DISCARDED}} = I_1 + I_2$$



- USE STANDARD INTEGRATION ON I_1 AND
- TRY TO OBTAIN A BOUND ON I_2 : i.e. $|I_2| < g(c)$
- CHOOSE c SUFFICIENTLY LARGE SO THAT $|I_2| < \text{TOLERANCE}$

$$\text{EG } I = \int_0^\infty e^{-x} \cos x dx = I_1 + I_2$$

$$|I_2| = \left| \int_c^\infty e^{-x} \cos x dx \right| < \int_c^\infty e^{-x} dx = e^{-c} < 10^{-6}$$

$$\Rightarrow c = -\ln 10^{-6} = 6 \ln 10$$

2. MAP TO A FINITE INTERVAL

CHOOSE MAP SO THAT $x^{-p} dx = dt$

$$\text{EG } p=2 \quad x^{-2} dx \sim dt \quad x^{-1} = t \quad x = t^{-1} \quad dx = -t^{-2} dt$$

$$-I = \int_0^a f\left(\frac{1}{t}\right) \frac{dt}{t^2}$$

$$\text{AS } t \rightarrow 0 \quad f\left(\frac{1}{t}\right) \frac{1}{t^2} \sim \left(\frac{1}{t}\right)^{-p} \cdot t^{-2} = t^{p-2} \quad p-2 > -1 \quad \text{SO INTEGRAL EXISTS}$$

3. USE SPECIALIZED GAUSS INTEGRATION RULES DEFINED ON INFINITE INTERVALS

eg GAUSS-LAGUERRE $[a, b] = [0, \infty)$ $w(x) = e^{-x}$

$$I = \int_0^\infty e^{-x} f(x) dx = \sum_{k=1}^n w_k f_k$$