Problem 1: (ODE Review) Find the general solutions of the following equations:

a. $(1 + x^2) y' + 2xy = \cot x$

b. $xy' = y \ln y$

c. $y'' - 5y' + 4y = 0$

d. $4y'' + 4y' + 2y = 0, y(0) = 2, y'(0) = 3$

e. $y'' - 6y' + 9y = 0,$

f. $2x^2y'' - xy' + 1y = Ax^{3/2} + Bx^2$

g. $x^2y'' - xy' + 5y = 0$

h. $x^2y'' - 3xy' + 4y = 0$

Problem 2: (Power series solution warm-up): Consider the following first order linear ODEs:

\begin{align*}
y' + (1 - 2x)y &= 0 \quad (1) \\
x'y' + (2 - x)y &= 0 \quad (2)
\end{align*}

a. Solve the differential equations (1) and (2) using the appropriate integrating factors.

b. Expand the solution to (1) as Taylor series about the point $x_0 = 0$. Expand the exponential in the solution to (2) as a power series.

c. Now for (1) assume a power series solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

obtain a recursion for the coefficients $a_n$. Use these recursions to determine the series representation of the solution. Compare this result to the series obtained in part b above.

d. Try using the same power series expansion (3) to solve (2). What happens?

e. Consider the following recursive strategy to generate an approximate solution to (2). Rewrite (2) as

$$xy' + 2y = xy$$

Now assuming $x \to 0$ and discarding the right hand side of (4), find a first order approximation $y_0$ as the solution to
\[ xy_0' + 2y_0 = 0 \]
Now substitute \( y_0 \) on the right side of (4) and solve for \( y_1 \)

\[ xy_1' + 2y_1 = xy_0 \]
Continue this process till you obtain \( y_2 \). How does \( y_2 \) compare with the series solution to (2) obtained in b? Can you use this series to motivate a modification to the series expansion (3) that would be appropriate to use to obtain a series solution to (2)?