Cubic Spline Interpolation

\[(1.2.3 \& 1.2.6)\]

- Given data \((x_1, y_1), \ldots, (x_n, y_n)\), fit the data with a piecewise defined function:

\[
f(x) = \begin{cases} 
    P_1(x) & \text{if } x_1 \leq x \leq x_2 \\
    P_2(x) & \text{if } x_2 \leq x \leq x_3 \\
    \vdots & \\
    P_{n-1}(x) & \text{if } x_{n-1} \leq x \leq x_n 
\end{cases}
\]

\[1\]

Such that

(a) \(f(x)\) is continuous (cts) at all points \(x_1, x_2, \ldots, x_n\).

(b) more smoothness – possibly

\[f'(x)\] exists everywhere \& cts

\[f''(x)\]  

...  

Def: Functions like above where each piece \(P_j(x)\) is a polynomial are called splines.

Our focus: We will require that

\[P_j(x)\] is a cubic polynomial

i.e.

\[P_j(x) = A_j x^3 + B_j x^2 + C_j x + D_j\]

for \(j = 1, \ldots, n-1\)
Rewrite:

\[ p_j(x) = A_j x^3 + B_j x^2 + C_j x + D_j \]

\[ j = 1, \ldots, n-1 \]

**Unknowns:** \( A_j, B_j, C_j, D_j \) for \( j = 1, \ldots, n-1 \)

\( \Rightarrow \boxed{4 (n-1) \text{ unknowns}} \)