Problem 1. Consider the following MATLAB code:

```matlab
>> B=vander(x);
>> p=size(B);
```

(a) If \( x = [1, 2, 3, 4] \), what is the value of the MATLAB variable \( p \)?

\[
p = [4, 4] \quad (\text{size}(A) = [m,n] \text{ when } A \text{ is an } m \times n \text{ matrix.})
\]

(b) Provide a vector for which \( B \) obtained as above would not be invertible.

\[
x = [0, 0, 0, 0] \quad x = [1, 1, 2, 3], \ldots
\]

(any \( x \) for which two or more entries are equal will do)

Problem 2. (12 points.) We wish to find a function \( f(x) \) that interpolates the points \((x_1, y_1), (x_2, y_2), (x_3, y_3), \) and \((x_4, y_4)\). We look for a function in the form

\[
f(x) = \begin{cases} 
  p_1(x); & x_1 \leq x \leq x_2 \\
  p_2(x); & x_2 \leq x \leq x_3 \\
  p_3(x); & x_3 \leq x \leq x_4 
\end{cases}
\]

where \( p_j(x) \) are polynomials of degree 5.

(a) How many unknowns are there to be determined to find such a function \( f(x) \)?

\( p_j \) is a poly. of deg. 5 \( \Rightarrow 6 \) coefficients to be determined for each \( j = 1, 2, 3 \). So, we need to determine 18 coefficients.

(b) The “smoothness conditions” (C1)-(C4) we imposed in class for cubic spline interpolation resulted in \( 4(n-1) \) equations where \( n \) is the number of data points. How many equations do we obtain in the interpolation problem above if we impose the same conditions (C1)-(C4)? (No need to write down the equations.)

Here \( n = 4 \), so we will still get \( 4(4-1) = 12 \) equations.

(c) How many solutions does the linear system of part (b) have? Justify your answer.

Assuming \( x_1, x_2, x_3, \) and \( x_4 \) are distinct, the system of 12 eqns will have infinitely many solutions.

This follows from the facts: (i) the system has at least one solution (given by the cubic polynomials we obtained in class) and (ii) we have more unknowns than equations, outlawing the possibility of a unique soln.