Math 307 Quiz 2 (Section 101)

Name: Student ID#: 

Problem 1. (4 points) Consider the following MATLAB code:

```matlab
>> A=vander(x);
>> p=size(A);
```

(a) If \( x = [1, 2, 3] \), what is the value of the MATLAB variable \( p \)?

\[
p = [3, 3] \quad (\text{size}(A) = [m,n] \text{ when } A \text{ is an } m \times n\text{ matrix})
\]

(b) Provide a vector \( x \) for which \( A \) obtained as above would not be invertible.

\[x = [0, 0, 0]; \quad x = [1, 1, 2], \ldots\]

(Any \( x \) for which two or more entries are equal will do)

Problem 2. (6 points.) We wish to find a function \( f(x) \) that interpolates the points \((x_1, y_1), (x_2, y_2), \text{ and } (x_3, y_3)\). We look for a function in the form

\[
f(x) = \begin{cases} 
p_1(x); & x_1 \leq x \leq x_2 \\
p_2(x); & x_2 \leq x \leq x_3
\end{cases}
\]

where \( p_j(x) \) are polynomials of degree 4.

(a) How many unknowns are there to be determined to find such a function \( f(x) \)?

\( p_j \) is a poly. of deg. 4 \( \Rightarrow \) 5 coefficients to be determined for each \( j=1,2 \). So, we need to determine 10 coefficients.

(b) The “smoothness conditions” (C1)-(C4) we imposed in class for cubic spline interpolation resulted in \( 4(n-1) \) equations where \( n \) is the number of data points. How many equations do we obtain in the interpolation problem above if we impose the same conditions (C1)-(C4)? (No need to write down the equations.)

Here \( n=3 \), so we will still get \( 4(3-1)=8 \) equations

(\text{Note that the number of eqns that (C1)-(C4) give does not depend on the degree of the polynomials}).

(c) How many solutions does the equation system of part (b) have? Justify your answer.

Assuming \( x_1, x_2, \text{ and } x_3 \) are distinct, the system of 8 eqns will have infinitely many solutions. This follows from the facts: (i) the system has at least one solution (given by the cubic polynomials we obtained in class), and (ii) we have more unknowns than equations, outruling the possibility of a unique soln.