Problem 1: Let \( P = (2, 3, 3) \) and \( Q = (3, 4, t) \), and \( O = (0, 0, 0) \) denotes the origin. Find the value of \( t \) so that \( \vec{OP} \) and \( \vec{OQ} \) are orthogonal.

\[
\vec{OP} = \langle 2, 3, 3 \rangle; \quad \vec{OQ} = \langle 3, 4, t \rangle.
\]

\[
\vec{OP} \cdot \vec{OQ} = 6 + 12 + 3t = 0 \quad \Rightarrow \quad 3t = -18 \quad \Rightarrow \quad t = -6
\]

Problem 2: Find the center and the radius of the sphere defined by

\[
x^2 + y^2 + z^2 + 4x - 2y - 4z + 4 = 0.
\]

**Solution:** Complete squares:

\[
x^2 + 4x + y^2 - 2y + z^2 - 4z + 4 = 0
\]

\[
\Rightarrow (x+2)^2 + (y-1)^2 + (z-2)^2 = 4 + 1 + 4 - 4 = 5
\]

So, center = \((-2, 1, 2)\)

radius = \(\sqrt{5}\)
Problem 3: Let \( \vec{u} = \langle -2, 1 \rangle \), \( \vec{v} = \langle 1, -2 \rangle \), and \( \vec{w} = \langle 3, 0 \rangle \) be vectors in \( \mathbb{R}^2 \).

(a) Compute the unit vector in the direction of \( \vec{w} \).

Call this vector \( \vec{a} \). Then

\[
\vec{a} = \frac{1}{\|\vec{w}\|} \vec{w} = \frac{1}{\sqrt{3^2 + 0^2}} \langle 2, 0 \rangle = \langle \frac{2}{\sqrt{13}}, 0 \rangle
\]

(b) Compute \( \vec{u} \cdot \vec{w} \). What does this value tell you about the angle between \( \vec{u} \) and \( \vec{w} \)? (NO NEED TO DETERMINE THE EXACT ANGLE!)

\[
\vec{u} \cdot \vec{w} = \langle 1, -2 \rangle \cdot \langle 3, 0 \rangle = 3
\]

Since \( \vec{u} \cdot \vec{w} > 0 \), the angle btw \( \vec{u} \) and \( \vec{w} \) is acute; i.e., it is btw \(-90^\circ \) and \(90^\circ \) (or \(-\frac{\pi}{2} \) and \(\frac{\pi}{2} \)).

(c) Plot the vectors \( \vec{u} \), \( \vec{v} \), \( \vec{u} - \vec{v} \), and \( -\frac{1}{2} \vec{u} \).

(They can start from any point you prefer. Label your plot.)