Ex: Sketch \( z = y^2 - x^2 \) (S)

(a) Sketch the intersection of \( S \) with \( z = \text{const. planes} \) (i.e. "\( z \)-traces" or "level curves" of \( S \))

(b) Sketch the \( y \)-traces (i.e. intersection of \( S \) with \( y = \text{const. planes} \))

Sol: (a) Set \( z = c \);
\[ y^2 - x^2 = c \]

. if \( c > 0 \), i.e. \( y^2 > x^2 \)
\[ \Rightarrow y^2 = x^2 + c \geq 0 \Rightarrow y = \pm \sqrt{x^2 + c} \]

. \( c = 0 \) \( \Rightarrow y = x \) and \( y = -x \)

. \( c < 0 \) \( \Rightarrow x^2 = y^2 - c \geq 0 \Rightarrow x = \pm \sqrt{y^2 - c} \)

. \( c = -2 \) \( \Rightarrow x = \sqrt{y^2 - c} \) \& \( x = -\sqrt{y^2 - c} \)

(b) \( y = k \) (constant)
\[ k^2 - x^2 = z \Rightarrow \text{parabola on } z = \text{plane} \]
Ex: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \)

- temperature: where and when \( T = f(x, y, z, t) \) & four var!
- volume of a gas \( V = f(P, T) \); two var!

In Math 200: mostly two or three variables: \( f(x, y) \) or \( f(x, y, t) \)

- One variable: \( y = f(x) \rightarrow \) graph is a curve \( y(x) : y = f(x), x \in \mathbb{R} \)
- Two variables: \( z = f(x, y) \)
  - \( y \) graph is \( \{(x, y, z) : z = f(x, y)\} \)
  - \( y \) graph is a surface

Three variables: \( w = f(x, y, z) \)
- \( z \) visualize by considering level surfaces
  - \( f(x, y, z) = k \)
  - \( k \)-level surface

Functions of several variables

In Math 100/101, we considered

- \( y = f(x) \) & function of one var.

In many applications, the quantity of interest may depend on more than one variable.
Important Definitions

- D: domain of f
  - \( \{ (x, y) \in \mathbb{R}^2 : f(x, y) \text{ is well-defined} \} \)
  - \( \text{well-defined} \)

- R: range of f
  - \( \{ z : z = f(x, y), (x, y) \in D \} \)
  - \( \text{set of all possible values f can take on!} \)

Graph of f
- \( \{ (x, y, z) : z = f(x, y), (x, y) \in D \} \)

Ex: \( f(x, y) = 2x + y \)
- \( D = \mathbb{R}^2 ; R = \mathbb{R} \)
- \( \text{graph}: z = 2x + y \)
- \( 2x + y - z = 0 \)
- \( \text{plane (with normal <2, 1, 1> etc.)} \)

(2) \( f(x, y) = \sqrt{1-x^2-y^2} \)

Domain: need \( 1 - x^2 - y^2 \geq 0 \)
- \( x^2 + y^2 \leq 1 \)
- \( \text{So, } D = \{ (x, y) : x^2 + y^2 \leq 1 \} \)

\[ R = [0, 1] \]

- \( \text{graph: Find level curves} \)
- \( c = \sqrt{1-x^2-y^2} \)
- \( x^2 + y^2 = 1 - c^2 \)
- \( z = c \)