Distances in 3d

(1) Point-to-point
\[ P_1 = (x_1, y_1, z_1), \quad P_2 = (x_2, y_2, z_2) \]
\[ d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \]

(2) Point-to-line
\[ \text{direction vector} \]
\[ P_0 : \text{(arbitrary) point on L} \]
\[ d = \| P_0 P - \text{proj}_{\vec{n}} P_0 P \| \]
ALT: \[ d = \| P_0 P \| \sin \theta \frac{\| \vec{n} \|}{\| \vec{u} \|} \]
\[ = \frac{\| P_0 P \times \vec{v} \|}{\| \vec{v} \|} \]

(3) Point-to-plane
\[ P_0 : \text{a point on the plane} \]
\[ \vec{n} : \text{normal vector of the plane} \]
\[ \text{dist} \ (P_1, E) = \| \text{proj}_{\vec{n}} \vec{P}_0 \vec{P} \| \]

(4) Line-to-line
\[ \text{given: } L_1, L_2 \ (\text{turn this into an example}) \]
\[ L_1 : \vec{r}_1(t) = \langle 0, 2, 0 \rangle + t \langle 0, 1, 2 \rangle \]
\[ L_2 : \vec{r}_2(s) = \langle 0, 1, 1 \rangle + s \langle 1, -1, 3 \rangle \]
E: the plane that
- contains \( L_2 \rightarrow \overrightarrow{v}_2 \parallel E \)
- \( L_1 \parallel E \rightarrow \overrightarrow{v}_1 \parallel E \)

\[ \Rightarrow \overrightarrow{n} = \overrightarrow{v}_1 \times \overrightarrow{v}_2 = \langle 0, 1, 2 \rangle \times \langle 1, -1, 2 \rangle = \langle 5, 2, -1 \rangle \]

Also: need a pt on \( E \rightarrow \) any pt on \( L_2 \) will do.
Say \( \overrightarrow{P} = (0, 2, 1) \)

\[ E: 5x + 2y - z = 1 \]

\[ \text{dist}(L_1, L_2) = \text{dist}(P_*, Q_*) = \text{dist}(\text{any pt on } L_1, E) \]

\[ \text{use formula from (3)} \]
\[ = \frac{1}{\sqrt{130}} \]

Quadric Surfaces & cylinders (10.1)

Cyber Cylinders: these are surfaces given by equations with one (or more) of \( x, y, z \) missing.
- \( x^2 + y^2 = 1 \) (\( z \) can be anything)
- \( x = \sin z \) (\( y \) is missing, so it can take any value)
- \( z + x = 1 \) (\( y \) is missing)

(Example 3.16 - p. 555)

Ex: \( x^2 + y^2 = 1 \)
\( S = \{ (x, y, z) : x^2 + y^2 = 1, 2 \in \mathbb{R} \} \)
For ex., \( (1, 0, 0) \in S \)
\( (1, 0, 100) \in S \)
In general, we use the word cylinder for any surface obtained:
- pick a curve $C$ on a plane
- extend parallel lines passing through $C$

**Quadric Surfaces**

Surfaces described by
$$Ax^2 + By^2 + Cz^2 + Dx y + E_{x z} + F_{y z} + G_{x t} H_{y t} I_{z t} + J = 0$$

**Includes:**
- planes ($A = B = ... = F = 0$)
- spheres ($A = B = C > 0, J < 0; ...$)
- parabolic cylinders, ... more!

**Special focus in the book**
- $A x^2 + B y^2 + z^2 = t$ \[t > 0\]
- $z = A x^2 + B y^2$ \[A, B > 0\]
- $z^2 = A x^2 + B y^2$ \[A, B > 0\]

*DO NOT MEMORIZE EQNS, NAMES, GRAPHS!*
Ex: Sketch $z = y^2 - x^2$ ($S$)

(a) Sketch the intersection of $S$ with $z = \text{const. planes}$ (i.e. "$z$-traces" or "level curves" of $S$)

(b) Sketch the $y$-traces (i.e., intersection of $S$ with $y = \text{const. planes}$)