Last time.

Ex. 1 \( \mathbf{r}_1(t) = \langle 1, 2, 3 \rangle + t \langle 1, 1, 1 \rangle \)
\( \mathbf{r}_2(s) = \langle 2, 1, 1 \rangle + s \langle 2, 1, 2 \rangle \)

Intersection? \( \mathbf{r}_1(t) = \mathbf{r}_2(s) \)

1) \( 1 + t = 2 + 2s \) \( \Rightarrow 3 - 2s = 1 = 0 \)
2) \( 2 + t = 1 + 2s \)
3) \( 3 + t = 1 + 2s \)

\( \Rightarrow \) lines are parallel. as \( \mathbf{v}_1 \parallel \mathbf{v}_2 \)

Ex. 2 \( \mathbf{r}_1(t) = \langle 1, 2, 3 \rangle + t \langle 1, 1, 1 \rangle \)
\( \mathbf{r}_2(s) = \langle -1, 0, 1 \rangle + s \langle 3, 3, 3 \rangle \)

Intersection?

1) \( 1 + t = -1 + 3s \) \( \Rightarrow t = 3s - 2 \) holds for all \( t \)
2) \( 2 + t = 3s \)
3) \( 3 + t = 1 + 3s \) \( \Rightarrow t = 3s - 2 \)

\( \Rightarrow \mathbf{r}_1 \) \& \( \mathbf{r}_2 \) are identical lines.

(3) \( \mathbf{r}_1(t) = \langle 1, 2, 3 \rangle + t \langle 1, 1, 1 \rangle \)
\( \mathbf{r}_2(s) = \langle 4, -1, 2 \rangle + s \langle -1, 2, 2 \rangle \)

Intersection? Set \( \mathbf{r}_1(t) = \mathbf{r}_2(s) \)

1) \( 1 + t = 4 - 5s \) \( \Rightarrow t = 1, s = 2 \)
2) \( 2 + t = -1 + 2s \)
3) \( 3 + t = 2 + 2s \)

\( \Rightarrow \) they intersect at one point

Point of intersection

\[ \mathbf{r}_1(1) = \langle 1, 2, 3 \rangle + \langle 1, 1, 1 \rangle = \langle 2, 3, 4 \rangle \]

\( P = (2, 3, 4) \) is the pt of intersection.

Exercise: give an example of two skew lines.
Planes (10.6)

\[ \mathbf{n} = \langle a, b, c \rangle \]

\[ \mathbf{PQ} \perp \mathbf{n} \]

\[ \mathbf{PA} = \langle x-x_0, y-y_0, z-z_0 \rangle \]

5. \[ \mathbf{PA} \cdot \mathbf{n} = 0 \quad \text{as} \quad \mathbf{PA} \perp \mathbf{n} \]

6. \[ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \]

Equation of the plane \( E \) (standard form)

OR: \[ ax + by + cz = d \quad (d = ax_0 + by_0 + cz_0) \]

general form

Ex: plane through \( P = (1, 2, 3) \) with \( \mathbf{n} = \langle 2, -3, 4 \rangle \)

\[ 2(x-1) - 3(y-2) + 4(z-3) = 0 \]

SOL: \[ 2(x-1) - 3(y-2) + 4(z-3) = 0 \]

OR \[ 2x - 3y + 4z = 8 \]

Remarks:

1. One can define a plane \( E \) by means of:

   i) a point on \( E \) and a normal vector \( \mathbf{n} \perp E \)

   ii) Three points \( P_1, P_2, P_3 \) then on \( E \)

   that are not collinear

   \[ \mathbf{n} = \mathbf{P_1P_2} \times \mathbf{P_1P_3} \]

   (iii) Two intersecting lines on \( E \)

2. Eqs of planes (like lines) are not unique: \( x + y + z = 1 \) & \( 2x + 2y + 4z = 2 \) → Same
Repeat:

\[ x + y + z = 1 \quad \text{and} \quad 2x + 2y + 2z = 2 \]

are the same plane.

3. Read off the normal vector easily:

\[ x + 2y + 3z = 5 \Rightarrow \vec{n_1} = \langle 1, 2, 3 \rangle \]
\[ 2x - 3y + 5z = 55 \Rightarrow \vec{n_2} = \langle 2, -3, 5 \rangle \]
\[ 7y + 3z - x = 1 \Rightarrow \vec{n_3} = \langle -1, 7, 3 \rangle \]

4. Two planes \( E_1 \) and \( E_2 \) are either (i) identical, (ii) parallel, or (iii) intersect at one line.

(a) If \( \vec{n_1} \parallel \vec{n_2} \), then (i) or (ii).

otherwise, (iii).

5. Angle btw \( E_1 \) \& \( E_2 \)

\[ = \text{angle btw } \vec{n_1} \text{ and } \vec{n_2} \]

Ex: \( E_1: 3x + 2y + z = 1 \)
\( E_2: x + y + 0.2 = 0 \)

(a) Find the intersection of \( E_1 \) \& \( E_2 \)

(b) \( \psi \) \& angle btw \( E_1 \) \& \( E_2 \)

Sol: (a) \( 3x + 2y + z = 1 \)
\( x + y = 0 \)

Note: if the planes are not parallel, then there will be one free variable, and the other two can be written in terms of that free variable.

Say \( x \) is the free variable here. Then set \( \overline{x = t} \), \( t \in \mathbb{R} \).

1. \( 3t + 2y + z = 1 \) \quad Solve \( \psi \) for \( y \& z \).
\( t + y = 0 \)
\( \Rightarrow y = -t \)

2. \( \Rightarrow \) into 1: \( 3t + 2(-t) + z = 1 \)
\( \Rightarrow z = 1 - t \)

So, the line of intersection:

\( x = 0 + t \)
\( y = 0 - t \)
\( z = 1 - t \)
\( \Rightarrow \langle x, y, z \rangle = \langle 0, 0, 1 \rangle \pm t \langle 1, -1, 1 \rangle \)
(2) angle $\theta$?

$\vec{n}_1 = \langle 3, 2, 17 \rangle$ ; $\vec{n}_2 = \langle 1, 1, 7 \rangle$

$\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{||\vec{n}_1|| \cdot ||\vec{n}_2||} = \frac{5}{\sqrt{14} \cdot \sqrt{12}} = \frac{5}{128}$

$\theta = \arccos \left( \frac{5}{\sqrt{128}} \right)$.

Distances in 3d