MATH 152 (2017-18 Winter 2)

Linear Systems

Our goals:
- Learn about vectors & matrices
  - geometric understanding
  - analytic skills
  - connections to applied problems
- learn how to solve linear systems of equations
- use complex numbers in the analysis of linear systems
- use eigenvalues/eigenvectors in the analysis of linear systems

Lots of applications in: Science, Engineering, Finance, Economics, ...

Ch 2 Vectors & Geometry

Review vectors, lines & planes, and linear systems from a geometric perspective.

2.2 Vectors

Vectors: quantities with both magnitude & direction.

\[ \vec{a} \rightarrow \vec{b} \rightarrow \vec{c} \]

magnitude of \( \vec{a} \): \( \| \vec{a} \| \) (length of \( \vec{a} \))

Why? Force, Velocity, electric field ... are vectors
What can we do with vectors?

**Basic operations**

VECTOR ADDITION:

\[ \vec{a} + \vec{b} \]

OR

\[ \vec{a} \]

\[ \vec{b} \]

SCALAR MULTIPLICATION:

\[ \lambda \vec{b} \]

\[ \vec{b} \]

\[ 2\vec{b} \]

\[ -\vec{b} \]

\[ \lambda \vec{b} \]

\[ \lambda \ll \vec{b} \ll \]

\[ |\lambda| \ll \vec{b} \ll \]

VECTOR SUBTRACTION:

\[ \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \]

Next, let's bring in numbers

To represent vectors using numbers, we first fix a coordinate system.

Then, we specify the components of the vector in the coordinate directions.

**R²: 2-dimensional Euclidean space**

Alternative way of writing \( \vec{a} \):

\[ \vec{a} = 3\hat{i} + 2\hat{j} \]

Standard basis vectors x in R²
$\mathbb{R}^3$: 3-dim. Euclidean space

As before: $\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$; $\hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

are the standard basis vectors in $\mathbb{R}^3$.

And: $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Similarly, $\mathbb{R}^n$: n-dim Euclidean space

$\vec{c} \in \mathbb{R}^n$: $\vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

We can denote a vector in many ways:

$\vec{a}$: what I use in class
$\mathbf{a}$: what the textbook uses
$\tilde{a}$: other popular notation
(a just regular a): when it's obvious

Another way:

$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

We work both with column vectors such as $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and row vectors such as $\vec{b} = [b_1, b_2, b_3]$. 

Some comments on notation & conventions
Addition / Subtraction (Scalar multiplication).

In $\mathbb{R}^2$: \[ \vec{a} = [a_1 \ a_2], \quad \vec{b} = [b_1 \ b_2] \]

Addition / Subtraction:
\[ \vec{a} + \vec{b} = [a_1 + b_1 \ a_2 + b_2] \]

Scalar Multiplication: $s \in \mathbb{R}$,
\[ s \vec{a} = [sa_1 \ sa_2] \]

Generalize to $\mathbb{R}^n$:
\[ \vec{a} + \vec{b} = [a_1 + b_1 \ \vdots \ a_n + b_n] \quad \text{or} \quad s \vec{a} = [sa_1 \ \vdots \ sa_n] \]