Math 105 (205): Quiz 3

Name:
Student Number:

Time: 15 minutes

1. [3 pts] Evaluate \( \int_0^{\pi/4} x \cos 2x \, dx \).

2. [3 pts] Evaluate \( \cos^3 x \sin x \, dx \).

3. [4 pts] Evaluate \( \int \frac{dx}{\sqrt{3 - 2x - x^2}} \).

\[ \int_0^{\pi/4} x \cos 2x \, dx = \frac{1}{2} \sin 2x \bigg|_0^{\pi/4} - \int_0^{\pi/4} \frac{1}{2} \sin 2x \, dx \]

\[ \begin{bmatrix} u(t) = x \\ u'(t) = 1 \end{bmatrix} \quad \begin{bmatrix} v(t) = \cos 2x \\ v'(t) = -\sin 2x \end{bmatrix} = \left( \frac{1}{2} \pi - 0 \right) + \frac{1}{4} \cos 2x \bigg|_0^{\pi/4} \]

\[ = \frac{\pi}{8} + \frac{1}{4} \cos \frac{\pi}{2} - \frac{1}{4} \cos 0 = \frac{\pi}{8} - \frac{1}{4} \]

\[ \int \cos^3 x \sqrt{\sin x} = \int \cos x (1 - \sin^2 x) \sqrt{\sin x} \, dx \]

\[ = \int \cos x (\sqrt{\sin x} - \sqrt{\sin x}) \, dx \]

Set \( u = \sin x \), \( du = \cos x \, dx \)

\[ = \int u^{1/2} - u^{5/2} \, du = \frac{2}{3} u^{3/2} - \frac{2}{7} u^{7/2} + C \]

\[ = \frac{2}{3} \sin^{3/2} x - \frac{2}{7} \sin^{7/2} x + C \]

\[ 3 - 2x - x^2 = 4 - (x^2 + 2x + 1) = 4 - (x + 1)^2 \]

So: set \( x + 1 = 2 \sin \theta \), \( dx = 2 \cos \theta \, d \theta \)

\[ \int \frac{dx}{\sqrt{3 - 2x - x^2}} = \int \frac{2 \cos \theta \, d \theta}{\sqrt{4 - 4 \sin^2 \theta}} \]

\[ = \int \frac{2 \cos \theta}{2 \cos \theta} \, d \theta = \theta + C = \arcsin \left( \frac{x + 1}{2} \right) + C \]