Question 1 (10 pts.)
Find the volume $V$ of the intersection of the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 2x$.

Answer: We use cylindrical coordinates (with respect to the $y,z$ plane):

\[
x = x \\
y = r \cos \theta \\
z = r \sin \theta.
\]

We have

\[
V = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_{\sqrt{1-r^2}}^{\sqrt{1-r^2} + 1} r \, dx \, dr \, d\theta = \int_0^{2\pi} \int_0^{\sqrt{2}} r(2\sqrt{1-r^2} - 1) \, dr \, d\theta \\
= -\frac{1}{2} \int_0^{2\pi} \int_1^{\sqrt{u}} (2\sqrt{u} - 1) \, du \, d\theta \\
= -\frac{1}{2} \int_0^{2\pi} \left( \frac{4}{3} u^{3/2} - u \right)_{u=1}^{u=\sqrt{u}} \, d\theta = \frac{5}{12}\pi
\]