Problem 1. Parameterize the curve \( y = \frac{2}{3}x^{3/2} \) with respect to arc length in the direction of increasing \( x \).

Problem 2. For a curve \( r(t) \), find an expression for \( \frac{dN}{dt} \cdot T \) in terms of \( |r'(t)| \) and curvature \( \kappa \). Here, \( N \) is the principal normal vector, and \( T \) is the unit tangent vector (hint: differentiate \( N \cdot T \)).

Problem 3. Calculate the arc length of the curve parameterized by \( x(t) = \cos^3(t) \), \( y(t) = \sin^3(t) \); \( 0 \leq t \leq \pi \).

Problem 4. Let \( C \) be the intersection of the cylinder \( x^2 + y^2 = 9 \) and the plane \( x + z = 0 \). We are looking for the points where the curvature is maximal.

1. Can you guess before doing any calculation how many points you are going to find? Explain.
2. Give a parametrization of \( C \) of the form \( r(t) = x(t)i + y(t)j + z(t)k \).
3. Find the points where the curvature is maximal.
4. Let \( u := \frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}k \), \( v := j \), \( w := \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}k \).
   Write \( r(t) \) in the form \( a(t)u + b(t)v + c(t)w \)
   where \( a(t) \), \( b(t) \) and \( c(t) \) are functions you have to determine.
5. Draw the curve \( C \) in the system of coordinates \((u, v, w)\).

Problem 5. Consider the ellipse \( x^2 + 2y^2 = 18 \). What is the equation of the osculating circle at point \((0,3)\)? Draw the curve and this osculating circle. To compute the normal vector, you can either use the formula learned in class (which might be complicated) or use the geometric description of this vector and figure out its coordinates.

Problem 6. (1) Calculate the work done by the vector field \( F(x, y) = (xy, 3y^2) \) on a particle moving
   (a) along the path \( r(t) = (11t^4, t^3) \) from \( t = 0 \) to \( t = 1 \);
   (b) along the line segment from \((0,0)\) to \((11,1)\).
2. Is the vector field \( F \) conservative?
Problem 7. Let $a \in \mathbb{R}$. Consider the vector field
\[\mathbf{F}(x, y, z) = (\cos(x) + (a - 3)y, 2 + \cos(y), e^z).\]

1. Find the value of $a$ for which the vector field is conservative and give $f$ such that $\mathbf{F} = \nabla f$.
2. For the value of $a$ you found in (1), compute the line integral of $\mathbf{F}$
   (a) along the line segment from $\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$ to $\left(\pi, \pi, 2\right)$;
   (b) along the curve $\mathbf{r}(t) = (\cos(t) + 1, \sin(t), 3\cos(t) + 4\sin(t))$ with $0 \leq t \leq 2\pi$;
   (c) along the spiral curve $\mathbf{r}(t) = (t, \cos(t), \sin(t))$ with $0 \leq t \leq 3\pi$.
3. Compute the integral
   \[\int_C (\cos(x) + 7y)dx + (2 + \cos(y))dy + e^zdz\]
   where $C$ is the curve (b) above with as little calculation as possible.