PRACTICE SET 2

SECTION 4.1

Exercise 4. Show that if \( a \) is an even integer, then \( a^2 \equiv 0 \mod 4 \), and if \( a \) is an odd integer, then \( a^2 \equiv 1 \mod 4 \).

Exercise 30. Show by mathematical induction that if \( n \) is a positive integer, then \( 4^n \equiv 1 + 3n \mod 9 \).

Exercise 36. Find the least positive residues modulo 47 of each of the following integers.

(a) \( 2^{32} \)
(b) \( 2^{47} \)
(c) \( 2^{200} \)

SECTION 4.2

Exercise 2. Find all solutions of each of the following linear congruences.

(a) \( 3x \equiv 2 \mod 7 \)
(b) \( 6x \equiv 3 \mod 9 \)
(c) \( 17x \equiv 14 \mod 21 \)
(d) \( 15x \equiv 9 \mod 25 \)

Exercise 6. For which integers \( c, 0 \leq c < 30 \), does the congruence \( 12x \equiv c \mod 30 \) have solutions? When there are solutions, how many incongruent solutions are there?

Exercise 8. Find an inverse modulo 13 of each of the following integers.

(a) 2
(b) 3
(c) 5
(d) 11

Exercise 10.

(a) Determine which integers \( a \), where \( 1 \leq a \leq 14 \), have an inverse modulo 14.
(b) Find the inverse of each of the following integers from part (a) that have an inverse modulo 14.
Section 4.3

Exercise 2. Find an integers that leaves a remainder of 1 when divided by either 2 or 5, but that is divisible by 3.

Exercise 4. Find all the solutions of each of the following systems of linear congruences.

(a) \( x \equiv 4 \pmod{11} \)
    \( x \equiv 3 \pmod{17} \)
    \( x \equiv 1 \pmod{2} \)
(b) \( x \equiv 2 \pmod{3} \)
    \( x \equiv 3 \pmod{5} \)
    \( x \equiv 0 \pmod{2} \)
    \( x \equiv 0 \pmod{3} \)
(c) \( x \equiv 1 \pmod{5} \)
    \( x \equiv 6 \pmod{7} \)

Exercise 22. An ancient Chinese problem asks for the least number of gold coins a band of 17 pirates could have stolen. The problem states that when the pirates divided the coins into equal piles, 3 were left over. When they fought over who should get the extra coins, one of the pirates was slain. When the remaining pirates divided the coins into equal piles, 10 coins were left over. When the pirates fought again over who should get the extra coins, another pirate was slain. When they divided the coins into equal piles again, no coins were left over. What is the answer to this problem?

Section 5.1

Exercise 2. Determine the highest power of 5 that divides each of the following positive integers.

(a) 112, 250
(b) 4,860, 625
(c) 235, 555, 790
(d) 48,126, 953, 125

Exercise 4. Which of the following integers is divisible by 11?

(a) 10,763, 732
(b) 1,086, 320, 015
(c) 674, 310, 976, 375
(d) 8,924, 310, 064, 537

Exercise 22. An old receipt has faded. It reads 88 chickens at a total of \$x.4.2y\), where \(x\) and \(y\) are unreadable digits. How much did each chicken cost?