Math 256. Sample Midterm exam.

No formula sheet, books or calculators!

**Part I**

Circle what you think is the correct answer. +3 for a correct answer, −1 for a wrong answer, 0 for no answer.

1. The ODE $y' - yp(x) = 0$, has the solution,
   
   (a) $CJ$ (b) $J + C$ (c) $J - C$ (d) $C/J$ (e) None of the above,
   
   where $C$ is a constant and $J = \exp[-\int p(x)dx]$.

2. The ODE $y' + f(x)/y = 0$, has the solution,
   
   (a) $\pm [C + 2\int f(x)dx]^{1/2}$ (b) $\pm [C - 2\int f(x)dx]^{1/2}$ (c) $\pm [C + \frac{1}{2}\int f(x)dx]^2$
   
   (d) $\pm [C - \frac{1}{2}\int f(x)dx]^2$ (e) None of the above,
   
   where $C$ is a constant.

3. The ODE $y'' - 4y' + 5y = 0$, has the solution,
   
   (a) $e^{2x}(A \cos x + B \sin x)$ (b) $e^{-2x}(A \cos x + B \sin x)$ (c) $Ae^{2x}\cos(2x + B)$
   
   (d) $Ae^{x}\cos(x + B)$ (e) None of the above,
   
   where $A$ and $B$ are constants.

4. The ODE $y'' + y' + 2y = 4x^2$, has the particular solution,
   
   (a) $2x^2 - 2x + 1$ (b) $2x^2 - 2x - 1$ (c) $2x^2 + 2x + 1$
   
   (d) $2x^2 + 2x - 1$ (e) None of the above.

**Part II**

Answer in full (i.e. give as many arguments, explanations and steps as you think is needed for a normal person to understand your logic). Answer as much as you can; partial credit awarded.

1. Define the integrating factor for the first-order ODE, $y' + yp(x) = q(x)$.
   
   Hence $(1 - x^2)y' - xy = \sqrt{1 - x^2}(1 + x^2)^2$ with $y(0) = 0$.

2. Solve the ODE,
   
   $y'' - 4y' + 4y = e^{\lambda x}$, \hspace{1cm} y(0) = y'(0) = 0,
   
   for (a) $\lambda \neq 2$, and (b) $\lambda = 2$. 