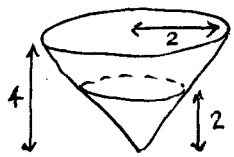
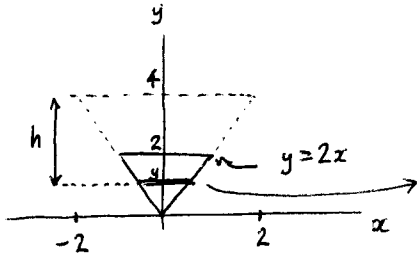
SCIE 001 Math
Webwork A12 #10
2014 Feb 14

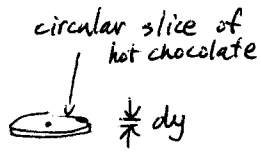
A tank in the shape of an inverted right circular cone has height 4 m and radius 2 m. It is filled with 2 m of hot chocolate. Find the work required to empty the tank by pumping the hot chocolate over the top of the tank. The (spatial, mass) density of hot chocolate is $\delta = 1040\ \text{kg}\cdot\text{m}^{-3}$.

Use SI units throughout.



Cross-section ↪





Horizontal infinitesimally thin "slice" of hot chocolate at height y is circular, with radius $x = \frac{1}{2}y$ and thickness dy

volume $dV = \pi x^2\,dy = \pi\left(\frac{1}{2}y\right)^2 dy = \frac{\pi}{4}y^2\,dy$ m^3

mass $dm = \delta\,dV = \frac{\pi\delta}{4}y^2\,dy$ kg

Work done to lift this mass dm against gravity a distance

$$h = \overset{\text{top}}{4} - \overset{\text{bottom}}{y} \quad \text{m}$$

is

$$dW = (dm)\,g\,h = \left(\frac{\pi\delta}{4}y^2\,dy\right) g\,(4-y) \quad \text{J}$$
$$= \frac{\pi\delta g}{4}(4-y)y^2\,dy$$

Total work done (note that the hot chocolate is from $y=0$ to $y=2$; no work is done if there isn't any hot chocolate to lift) is

$$W = \int_{y=0}^{y=2} dW = \frac{\pi\delta g}{4}\int_0^2 (4-y)y^2\,dy$$
$$= \frac{\pi\delta g}{4}\int_0^2 (4y^2 - y^3)\,dy$$
$$= \frac{\pi\delta g}{4}\left(\frac{4}{3}y^3 - \frac{1}{4}y^4\right)\Big|_0^2$$
$$= \frac{5}{3}\pi\delta g \quad \text{Joules}$$
$$\approx \frac{5}{3}\pi(1040)(9.8)$$
$$\approx 53365.2 \quad \text{J}$$