

SCIE 001 MATHEMATICS ASSIGNMENT 11 (Due 10:00 am Feb. 7, 2014)

There are two parts to this assignment. The first part is on WeBWorK and is due by 10:00 am on Fri. Feb. 7. The second part consists of the questions on this page. This assignment is due by 10:00 am on Fri. Feb. 7. For these questions, you are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typeset or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. Make the substitution $u = \sqrt[12]{x}$ and then evaluate

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}.$$

2. Suppose the (open) first quadrant of the xy -plane (i.e. $x > 0$, $y > 0$) represents water, viewed from above. The rest of the xy -plane represents solid land. You are initially at the origin, holding the end of a rope of length $L > 0$ which is tied to a boat at the point $(L, 0)$ in the xy -plane. Then you walk along the positive y -axis, keeping the rope straight and taut (let us assume you are strong enough to pull the boat, the rope doesn't stretch, and you don't fall into the water trying to balance on the y -axis). The boat follows a path $y = f(x)$, with the property that the rope is always tangent to the path of the boat (figure on p. 534). (If having the boat initially at $(L, 0)$ doesn't offend your sensibilities, then don't worry about it; for those of you that worry about such things, you can think of the boat initially at the point (L, ε) , where $\varepsilon > 0$ is much smaller than L .)

- (a) Show that

$$f'(x) = \frac{dy}{dx} = -\frac{\sqrt{L^2 - x^2}}{x}.$$

- (b) Determine the function $y = f(x)$.