

SCIE 001 MATHEMATICS ASSIGNMENT 2 (Due 10:00 am Mon Oct. 7, 2013)

There are two parts to this assignment. The first part is on WeBWorK and is due at 10:00 am on Mon. Oct. 7. The second part consists of the questions on this page. This assignment is due by 10:00 am on Mon. Oct. 7. For these questions, you are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typeset or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. Let

$$f(x) = \begin{cases} |x|^{3/2} & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ x^{3/2} \sin\left(\frac{1}{x}\right) & \text{if } x > 0. \end{cases}$$

For each of the following statements, determine if it is true or false. If true, give a proof. If false, give a counterexample (and justify it).

- f is continuous at every point.
 - f is differentiable at every point.
 - f' is continuous at every point.
2. Look up the definitions of left- and right-hand limits, on p. 113. Let f be a function defined on an open interval that contains 0.
- If f is an *even* function, i.e., $f(-x) = f(x)$ for all x , and if $\lim_{x \rightarrow 0^+} f(x) = L$, prove using an ε - δ argument that $\lim_{x \rightarrow 0^-} f(x) = L$. If this f is also differentiable at 0, use the definition of the derivative to determine $f'(0)$.
 - If f is an *odd* function, i.e., $f(-x) = -f(x)$ for all x , and if $\lim_{x \rightarrow 0^+} f(x) = L$, prove using an ε - δ argument that $\lim_{x \rightarrow 0^-} f(x) = -L$. If this f is also continuous at 0, use the definition of continuity to determine $f(0)$.
3. Look up the definition of a function that is continuous on an interval, on p. 120. For $n = 1, 2, 3, \dots$, define

$$f\left(\frac{1}{n}\right) = \frac{(-1)^{n-1}}{n}$$

so $f(1) = 1$, $f(1/2) = -1/2$, $f(1/3) = 1/3$, $f(1/4) = -1/4$, etc. Then for $1/(n+1) < x < 1/n$ define $f(x)$ so that the graph of $y = f(x)$ is the straight line segment between the points $(1/(n+1), f(1/(n+1)))$ and $(1/n, f(1/n))$. Finally, define $f(0) = 0$.

- Sketch the graph of $y = f(x)$, $0 \leq x \leq 1$.
- Determine a formula for $f(x)$, when $1/(n+1) < x < 1/n$.
- Show that f is continuous on the closed interval $[0, 1]$.
- Show that f is not differentiable at an infinite number of points in the open interval $(0, 1)$.