HW 3 due 9:00 am Fri Jan 26

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.pdf or .jpg format ONLY

.scans preferred (or LaTeX-produced pdf)

Photos ok if legible.

EN'TIRE ASSIGNMENT as Q1

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Example 3.4.B

\[ 2t^2 y'' + 3ty' - y = 0 \quad y(1) = 3 \]
\[ y'(1) = 0 \]

(b) cont.

We have checked \( y_1(t) = t^{-1} (= \frac{1}{t}) \) is a solution.

Reduction of order method:

Guess \( y(t) = v(t) y_1(t) (= v(t) t^{-1}) \)

Check plug it in and calculate…
\[ y = v y_1 \]
\[ y' = v' y_1 + v y_1' \]
\[ y'' = v'' y_1 + 2 v' y_1' + v y_1'' \]
\[ 2 t^2 \left( v'' y_1 + 2 v' y_1' + v y_1'' \right) + 3 t \left( v' y_1 + v y_1' \right) - (v y_1) \equiv 0 \]
\[ 2 t^2 y_1 v'' + (4 t^2 y_1' + 3 t y_1) v' \]
\[ + (2 t^2 y_1'' + 3 t y_1' - y_1) v \equiv 0 \]
\[ = 0 \text{ because } y_1(t) \text{ is a solution} \]
\[ 2 t^4 t^{-1} v'' + (4 t^4 (-t^{-2}) + 3 t t^{-1}) v' \equiv 0 \]
\[ 2 t v'' - v' = 0 \]
\[ 2 t (v')' - (v') = 0 \quad \text{1st-order linear eqn. for } v' \]

Solve by integrating factor method

\[ (v')' - \frac{1}{2t} (v') = 0 \]

Integrating factor \( \mu(t) = e \int (-\frac{1}{2t}) \, dt \]
\[ = e^{-\frac{1}{2} \ln t} = t^{-\frac{1}{2}} \]
\[ t^{-\frac{3}{2}} (v')' - t^{-\frac{3}{2}} \frac{1}{2t} (v') = t^{-\frac{3}{2}} \cdot 0 \]

\[ \frac{1}{2} t^{-3/2} \to (t^{-\frac{3}{2}} (v'))' = 0 \]

\[ t^{-\frac{3}{2}} (v') = k_2 \text{ (arb. constant)} \]

\[ v' = k_2 t^{-\frac{3}{2}} \]

\[ v(t) = \int k_2 t^{-\frac{3}{2}} \, dt = k_2 \cdot \frac{2}{3} t^{\frac{3}{2}} + c_1 \]

\[ = c_1 + c_2 t^{\frac{3}{2}} \]

\[ y(t) = v(t) \quad y_1(t) = (c_1 + c_2 t^{\frac{3}{2}}) t^{-1} \]

Take \( c_2 = 1, c_1 = 0 \) : \( y_2(t) = t^{\frac{1}{2}} \)

This is a solution (we have derived it, but can always check)

Fundamental set?

\[ \begin{cases} y_1 \text{ is a solution} & \checkmark \\ y_2 \text{ is a solution} & \checkmark \\ W[y_1, y_2] \neq 0 \quad ? \text{ need to check} \end{cases} \]
\[ W[y_1, y_2](t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t^{-1} & t^{-1/2} \\ -t^{-2} & \frac{1}{2} t^{-3/2} \end{vmatrix} = \frac{1}{2} t^{-3/2} + t^{-3/2} = \frac{3}{2} t^{-3/2} \]

\[ \neq 0 \text{ on } (0, \infty) \]

\[ y_1(t) = \frac{1}{t}, \quad y_2 = \sqrt{t} \quad \text{is a fundamental set on } 0 < t < \infty \]

(c) Solve initial value problem.

General solution is \[ y(t) = c_1 t^{-1} + c_2 t^{-1/2} \]

\[ y'(t) = -c_1 t^{-2} + \frac{1}{2} c_2 t^{-1/2} \]
\[ y(1) = c_1 + c_2 = 3 \]
\[ y'(1) = -c_1 + \frac{1}{2}c_2 = 0 \]
\[ c_1 = 1, \ c_2 = 2 \]

Solu. of initial value problem is:
\[ y = \phi(t) = t^{-1} + 2t^{-2} \]
\[ = \frac{1}{t} + 2\sqrt{t} \quad (0 < t < \infty) \]

3.5 **Nonhomogeneous equations; method of undetermined coefficients**

(1) \[ y'' + p(t)y' + q(t)y = g(t) \]
\[ L[y] \]

Corresponding homogeneous eqn. is

(2) \[ L[y] = 0 \]
Theorem 3.5.2  General solution of (1) is

\[ y = \phi(t) = c_1 y_1(t) + c_2 y_2(t) + Y(t) \]

where \( c_1 y_1(t) + c_2 y_2(t) \) is the general solution of the corresponding homogeneous equation (2), and \( Y \) is any solution of the non-homogeneous equation (1).

Method of undetermined coefficients:

If \( L[y] \) has constant coefficients and \( g(t) \) is ‘nice’, we can find \( Y(t) \) by guessing its form:

\[ ay'' + by' + cy = g(t) \]

If necessary \( g(t) = g_1(t) + \ldots + g_m(t) \) where each \( g_i(t) \) is ‘nice’

Table 3.5.1 (p.139) [10th ed. p.182]