NOTE: Textbooks written for U.S.A. markets (such as our textbook) often use FPS (foot-pound-second) units. In the FPS (or “Imperial”) system of units, the unit of **length** is the foot (ft), and 1 ft = 12 inches (in). [The SI or “metric” unit of length is the metre (m).] The unit of **time** is the second (s) [the same as the SI unit of time]. The FPS unit of **force** is the pound (lb) [the SI unit of force is the Newton (N)]. **Weight** is a type of force, so it is measured in pounds, weight = mg where g is acceleration due to gravity (g = 32 ft · s\(^{-2}\) in FPS units, approximately, at the surface of the earth) and m is the mass, measured in FPS units of **slugs** or lb · s\(^2\) ft\(^{-1}\) [the SI unit of mass is the kilogram (kg)]. In FPS units, “an object weighs 5 lb” means mg = 5 lb, where g = 32 ft·s\(^{-2}\), so m = \(\frac{5}{32}\) slugs; a mass of \(\frac{5}{32}\) slugs weighs 5 lb. [But in SI units, a mass of 5 kg weighs mg = (5 kg)(9.8 m · s\(^{-2}\)) = 49 N (approximately, at the surface of the earth).]

1. A mass that weighs 3 lb stretches a spring 3 in. The mass is pushed upwards from its equilibrium position, contracting the spring a distance of 1 in, then set in motion with a downwards velocity of 2 ft/s. If the motion is undamped, find the natural frequency \(\omega_0\), amplitude \(R\) and phase \(\delta\) of the downwards displacement \(u(t) = R \cos(\omega_0 t - \delta)\).

2. A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with damping constant of 2 lb · s/ft. If the mass is set in motion from its equilibrium position with a downwards velocity of 3 in/s, find its downwards displacement at any time \(t \geq 0\). Write the displacement in the form \(u(t) = Re^{\lambda t} \cos(\mu t - \delta)\) and determine \(R, \lambda, \delta\) and the quasi-frequency \(\mu\), using exact expressions.

3. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of 10 sin\((t/2)\) N and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial downwards velocity of 3 cm/s:
   (a) Find the position of the mass at any time \(t \geq 0\).
   (b) Identify the transient and steady-state parts of the solution.
   (c) If the given external force is replaced by a force of 2 cos(\(\omega t\)) with forcing frequency \(\omega\), find the value of \(\omega\) for which the amplitude of the forced response (or “steady-state”) is the maximum possible.

4. An undamped mass-spring system with a mass that weighs 6 lb and a spring constant of 12 lb/ft is suddenly set in motion at \(t = 0\) by an external force of 4\(\cos(7t)\) lb. Determine the displacement of the mass at any time \(t \geq 0\), and write the displacement as a product of two trigonometric functions of different frequencies. Sketch the graph of the position versus \(t\), showing the amplitude modulation with the slow frequency.