MATH 200, Homework 6

1. (a) Find all critical points of the function

\[ f(x, y) = 4x^2 + 3y^2 + 4xy - 4x + 2y \]

(b) Use the second derivative test to classify the critical points as either min, max, or a saddle point. If it is a max/min, is a global max/min? Briefly explain why/why not.

2. (a) Use the method of Lagrange multiplier to find the extremum of the function \( f(x, y, z) = 2x + y + z \) subject to the constraint \( xyz = 1 \) and \( x, y, z \geq 0 \).

(b) The extremum you found, does it correspond to a maximum or a minimum?

3. Find the minimum and maximum of the function \( f(x, y) = x^2 + 2y^2 - 2x - 4y \) inside a triangular domain \( D = \{(x, y) : x + y \leq 4, \ x \geq 0, y \geq 0\} \). (you need to check the boundaries and interior for max/min).

4. You want to make box with volume 16 m\(^3\). The material for the sides costs half of what the material for the top or bottom cost. Find the dimensions that will minimize the cost.

5. The plane \( x + y + 2z = 2 \) intersects the surface \( z = x^2 + y^2 \). Find the point on the intersection of these two surfaces which is (a) closest to z-axis and (b) furthest away from the z-axis.