MATH 200
Formulas (for Midterm Test 2)

A) \( u \) is a differentiable function of \( x_1, x_2, \ldots, x_n \) and each \( x_j \) is a differentiable function of \( t_1, t_2, \ldots, t_m \):

\[
\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2} \frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n} \frac{\partial x_n}{\partial t_i}
\]

B) \( F(x, y, z) = 0 \) defines \( z \) implicitly as a differentiable function of \( x, y \):

\[
\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z},
\]

C) \( f \) is a differentiable function of two or three variables, \( \mathbf{u} \) is a unit vector (\( \|\mathbf{u}\| = 1 \)):

\[
\mathbf{D}u = \nabla f \cdot \mathbf{u}, \quad \nabla f = \langle f_x, f_y \rangle \text{ or } \nabla f = \langle f_x, f_y, f_z \rangle
\]

D) \( F \) is a differentiable function of three variables:

\[
F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0
\]

E) \( f \) is a function of two variables with continuous second partial derivatives: three cases (a) \( D > 0 \) and \( f_{xx} > 0 \); (b) \( D > 0 \) and \( f_{xx} < 0 \); (c) \( D < 0 \), where

\[
D = f_{xx}f_{yy} - (f_{xy})^2
\]

F) \( f \) and \( g \) are differentiable functions of two or three variables:

\[
\nabla f = \lambda \nabla g, \quad g = k
\]

G) \( f \) is an integrable (e.g. continuous) function of two variables, \( D \) is a two-dimensional region:

\[
\int \int_D f(x, y) \, dA = \int_a^b \int_{g_2(x)}^{g_1(x)} f(x, y) \, dy \, dx = \int_c^d \int_{h_2(y)}^{h_1(y)} f(x, y) \, dx \, dy
\]

H) \( x = r \cos \theta, \ y = r \sin \theta, \ r^2 = x^2 + y^2, \ \tan \theta = y/x \):

\[
\int \int_D f(x, y) \, dA = \int_0^\beta \int_{h_2(\theta)}^{h_1(\theta)} f(r \cos \theta, r \sin \theta) \, r \, dr \, d\theta
\]

I) \( \rho \) is an integrable (e.g. continuous) function of two variables, \( D \) is a two-dimensional region:

\[
m = \int \int_D \rho(x, y) \, dA, \quad \bar{x} = \frac{M_y}{m} = \frac{1}{m} \int \int_D x \rho(x, y) \, dA, \quad \bar{y} = \frac{M_x}{m} = \frac{1}{m} \int \int_D y \rho(x, y) \, dA,
\]

J)

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