Midterm review session

Monday 6:00-8:00 pm  TBLC 182.
Section 003: Review session  Friday 5:30 - 7:30 pm  HENN 205

Last time: Geometric problems with derivatives...
Important fact (NEVER FORGET)

\( f'(a) \) is the slope of the tangent line of the graph of \( f(x) \) at \((a, f(a))\).

equation of tangent line: \[ y - f(a) = f'(a)(x-a) \]

Also: RECALL

\( f'(a) \): instantaneous rate of change of \( f(x) \) at \( x=a \).

If \( f(t) \) is temperature at time \( t \)
\( f'(t) \) is instantaneous rate of change of temperature at time \( t \) ....

Differentiability (NOT in the midterm).

\( f(x) \) is differentiable at \( x=a \) when
\[ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \] exists and is a number.
**FACT:** If \( f(x) \) is differentiable at \( x = a \) then \( f(x) \) is continuous at \( x = a \).

\[ \text{Diff} \implies \text{Cont.} \]

We saw: \( f(x) = |x| \) is not differentiable at \( x = 0 \).

**FACT:** If the graph of \( f(x) \) has a corner \((\forall)\) or \((\wedge)\) or a cusp \((\vee)\) or a "cor-cusp" \((\wedge)\) or \((\forall)\) at \( x = a \) then \( f(x) \) is not differentiable at \( x = a \).

**Example:**

- At \( x = -3 \), \( f(x) \) is not dif. because it is not cont.
- At \( x = 1 \), \( f(x) \) is not dif. because it has a cor-cusp \( \forall \).
- At \( x = 1.5 \), \( f(x) \) is not dif. because it has a corner \( \forall \).
- At \( x = 3 \), \( f(x) \) is not dif. because it has a cor-cusp \( \wedge \).
- At \( x = 4 \), \( f(x) \) is not differentiable because it has a cor-cusp \( \wedge \).
Exponential functions

We talked about $x^2$ a lot...
Now it's time to introduce some new functions...

$f(x) = 2^x$

What is this function?

Table of values

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2^1 = 2$</td>
</tr>
<tr>
<td>2</td>
<td>$2^2 = 4$</td>
</tr>
<tr>
<td>3</td>
<td>$2^3 = 8$</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>$2^0 = 1$</td>
</tr>
<tr>
<td>-1</td>
<td>$2^{-1} = \frac{1}{2}$</td>
</tr>
<tr>
<td>-2</td>
<td>$2^{-2} = \frac{1}{4}$</td>
</tr>
</tbody>
</table>

Plot them in a graph:

Graph of $f(x) = 2^x$
Observations from this graph:

Domain of $f(x) = 2^x$: All real numbers
$2^x > 0$ for all $x$
$2^x$ is NEVER zero (NO x-intercepts!!!)
But it gets closer and closer to 0 as $x$ becomes more and more negative
$2^x$ is becoming bigger and bigger as $x$ becomes bigger and bigger.

$y$-intercept: $(0, 2^0) = (0, 1)$.
$f(x) = 2^x$ is one to one.

We can also talk about $y(x) = 3^x$, $4^x$, $5^x$, ...

Q. Solve the equation $2^{x+1} = 1$.

We need $x + 1 = 0$ (Recall $2^0 = 1$)

\[\begin{align*}
x &= -1
\end{align*}\]

Q. Solve the equation $2^x = 4$.
We $2^x = 4$ is $2^x = 2^2$ so $x = 2$

Algebraic properties:

\[\begin{align*}
2^0 &= 1 \\
2^a \cdot 2^b &= 2^{a+b} \\
2^{-a} &= \frac{1}{2^a} \\
(2^a)^b &= 2^{ab}
\end{align*}\]
A. Solve the equation

\[ 4^{-2x} = \sqrt[3]{4} \]

\[ \Rightarrow \text{ We write } \quad 4^{-2x} = 4^{\frac{2}{3}} \]

\[ 4^{-2x} \cdot 4^{-\frac{2}{3}} = 1 \]

\[ 4^{-2x - \frac{2}{3}} = 1 \]

\[ -2x - \frac{1}{3} = 0 \]

\[ -2x = \frac{1}{3} \]

\[ x = -\frac{1}{6} \]