Friday September 16.

Last time:
- parallel - perpendicular lines
- functions; graph of a function.

\[ f(x) = \frac{1}{x-1} \]
\[ f(x) = x^3 + 1 \]

\[ \frac{d}{dx} y = 2x + b \quad \ldots \quad y = ax + b \]

\[ x\text{-intercept: The point at which the graph of the function intersects the } x\text{-axis.} \]
We find it by solving \( y = f(x) = 0 \).
In the line \( y = 2x + 1 \) it is \( (-\frac{1}{2}, 0) \).
y-intercept: The point at which the graph of the function intersects the y-axis. We find it by plugging in \( x = 0 \) as \((0, f(0))\). In the line \( y = 2x + 1 \), it is \((0, 1)\).

Some more complicated functions – quadratics

They have the form \( y = f(x) = ax^2 + bx + c \) for \( a, b, c \) constant real numbers; \( a \neq 0 \).

For example:
- \( f(x) = x^2 \)

\[ f(x) = x^2 + 2x + 1 \]

The graph looks like a parabola. To plot it, we need some points in it. Points that are "easy" to find:
- \( x \) and \( y \)-intercept.

Here:

**x-intercept**: Solving \( y = x^2 + 2x + 1 = 0 \)

\( (x + 1)^2 = 0 \)

\( x = -1 \).

Hence the point \((-1, 0)\) is in the graph.
y-intercept: Plug in $x = 0$
then we get the point $(0, f(0)) = (0, 1)$ which is also in the graph of the function.

Graph:

- Piece-wise functions:

We can also "combine" functions to get a "new" one; as in the example:

$$f(x) = \begin{cases} 
  x^2 + 2x + 1 & \text{if } x \leq 4 \\
  x - 1 & \text{if } x > 4
\end{cases}$$

Graph:
Another way to get more functions

Composition of functions

If $f: X \rightarrow Y$ is a function domain \rightarrow range.

and $g: Y \rightarrow Z$ is another function.

I create $g \circ f: X \rightarrow Y$ a new function;

$$(g \circ f)(x) = g(f(x)).$$

$$\begin{align*}
X & \xrightarrow{f} Y & \xrightarrow{g} Z \\
\forall x \in X & \mapsto f(x) \mapsto g(f(x))
\end{align*}$$

Example:

\[ f(x) = x^2 + 2x + 1 \]

A. Can you write it as a composition of two functions?

A. $\forall x \mapsto f(x) = x^2 + 2x + 1 = (x+1)^2$.

If \[ g(x) = x + 1 \]

$(h \circ g)(x) = h(g(x)) = h(x+1) = (x+1)^2$. \[ \square \]
More examples:

1. If \( f(x) = x - 5 \)
   \[ g(x) = \frac{1}{x} \]
   What is \( f \circ g \) ? \( g \circ f \) ?
   \[
   f \circ g(x) = f\left(\frac{1}{x}\right) = \frac{1}{x} - 5
   
   g \circ f(x) = g\left(x - 5\right) = \frac{1}{x - 5}
   
   \]

2. If \( f(x) = |x| \)
   \[ g(x) = x^2 - 1 \]
   What is \( f \circ g \) ? \( f(g(x)) = |x^2 - 1| \)
   This is also a piecewise function!
   \[
   f \circ g(x) = |x^2 - 1| = \begin{cases} 
   x^2 - 1 & \text{if } x^2 \geq 1 \\
   -(x^2 - 1) & \text{if } x^2 < 1
   \end{cases}
   
   = \begin{cases} 
   x^2 - 1 & \text{if } x \geq 1 \text{ or } x \leq -1 \\
   -x^2 + 1 & \text{if } -1 < x < 1
   \end{cases}
   
   \]
Note: Graph of \(|x| = f(x) = \begin{cases} x; x \geq 0 \\ -x; x < 0 \end{cases}\)

When is \(x^2 \geq 1\)?
we can either have \(x \geq 1\) (e.g., 5, 6, ...)
or \(x \leq -1\) (e.g., -5, -6, ...)
because \((a)^2 = (-a)^2\).

Some times you are given a table of values and are asked to draw a function.
It looks like this:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(\frac{8}{2})</td>
</tr>
<tr>
<td>3</td>
<td>(2\pi)</td>
</tr>
<tr>
<td>3.5</td>
<td>(9.2875)</td>
</tr>
<tr>
<td>(-1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(-2)</td>
<td>((-8))</td>
</tr>
</tbody>
</table>

keep in mind: Graph of \(x^3\)
Inverse function:

Remember... a function is a rule that takes \( x \) and does something to it. Sometimes, we can undo this by applying another function.

\[
\begin{align*}
  f & \quad \rightarrow \quad g \\
  x & \quad \mapsto \quad f(x) & \quad \mapsto \quad g(f(x)) = x.
\end{align*}
\]

This function (here \( g \)) is called the inverse of \( f \) and is denoted by \( f^{-1} \).

How does the graph of \( f^{-1} \) look like?

If I have \((x, f(x) = y)\) a point in the graph of \( f \), then \((y, x)\) (flip the coordinates) is in the graph of \( f^{-1} \).

Example - lines

\[ f(x) = ax + b. \]

\( a \neq 0 \)

To find the inverse, we set \( y = f(x) \) and solve for \( x \) in terms of \( y \) to get \( x = f^{-1}(y) \).