MATH 200 253 (Circle one)

Special Instructions:
No memory aids, calculators, or electronic devices of any kind are allowed on the test. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. Numerical answers should be left in calculator-ready form, unless otherwise indicated. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
[8] 1. Consider the function \( f(x, y) = \sqrt[3]{x} \cdot e^y \).

a) (2 marks) Is the function \( f(x, y) \) differentiable at the point \((1, 0)\)? Justify your answer.

b) (4 marks) Write down the differential \( df \) of the function \( f(x, y) \) at a generic point \((x, y)\), and at the point \((1, 0)\), using \( dx \) and \( dy \) as increments of the two variables.

c) (2 marks) Using the differential that you computed in b), find an approximate value for the number \( \sqrt[3]{0.97} e^{0.03} \).
[8] 2. Given the surface $S$ in space with equation $x^3z^2 + xy = 4$:

a) (2 marks) Find an equation of the tangent plane of the surface $S$ at the point $(1, 0, 2)$.

b) (3 marks) Assuming that the equation of the surface $S$ defines $z$ implicitly as a function of $x$ and $y$ around the point $(1, 0, 2)$, find the value of $\partial z / \partial x$ at the point $(x, y) = (1, 0)$. Your answer should be a number.

c) (3 marks) A person hiking on the surface $S$ is in the point $(1, 0, 2)$ and wants to move in a direction in which the air humidity $H(x, y, z) = xyz$ stays constant. Which direction should he head to? Your answer should be a vector with 3 components, and not necessarily of length 1.
[8] 3. Consider the lines given by parametric equations

\[ L_1 : \begin{cases} x = 1 \\ y = 1 \\ z = 1 + t \end{cases} \quad L_2 : \begin{cases} x = 0 \\ y = s \\ z = 0 \end{cases} \]

a) (2 marks) Write down the distance between a generic point on \( L_1 \) and a generic point on \( L_2 \) as a function \( f(s, t) \) of the two parameters \( s \) and \( t \).

b) (4 marks) Set \( g(s, t) = f(s, t)^2 \) to be the square of the function \( f \). Find the only critical point of the function \( g(s, t) \), and classify it (for example using the second derivative test).

c) (2 marks) Compute the value of the function \( f(s, t) \) on the critical point you found in b), and give a geometric interpretation of this value and of the critical point.
4.

a) (2 marks) Sketch the region $D = \{(x,y) \mid x \geq -1, \ y \geq 0, \ \text{and} \ y \leq 4 - x^2\}$ in the plane. Is it closed and bounded?

b) (6 marks) Find the absolute maximum and absolute minimum of the function

$$f(x,y) = xy - x + 1$$

in the region $D$. 
5. Consider the function \( f(x, y) = x^3 e^y \).

a) (5 marks) Using Lagrange multipliers, find maximum and minimum values of the function \( f(x, y) \) on the circle \( x^2 + y^2 = 4 \).

b) (3 marks) Check that all points \((0, y)\) with zero \( x\)-coordinate are critical points of the function \( f(x, y) \). Are they local maxima, local minima or saddle points?