The University of British Columbia
Midterm Examination - May 27, 2015

Closed book examination Time: 60 minutes

Last Name ___________________________ First _______________________

Student Number ___________________________ Signature _______________________

MATH 200 253 (Circle one)

Special Instructions:

No memory aids, calculators, or electronic devices of any kind are allowed on the test. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. Numerical answers should be left in calculator-ready form, unless otherwise indicated. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

• Each candidate must be prepared to produce, upon request, a UBCcard for identification.
• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
• Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
• Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

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1. Consider points \( A = (0,1,1), B = (2,1,0) \) and \( C = (3,1,1) \) in space, and call \( \pi \) the plane that passes through \( A, B \) and \( C \).

a) (3 marks) Find the area of the triangle with vertices the points \( A, B, C \).

b) (3 marks) Find all vectors orthogonal to the plane \( \pi \) and of length 16.

c) (2 marks) Determine all vectors \( \vec{v} = \langle x, y, z \rangle \) orthogonal to \( \overrightarrow{AB} \) and such that \( \overrightarrow{AB} \times \vec{v} = \langle 0, 5, 0 \rangle \).

d) (2 marks) Determine all values of the real parameter \( a \) for which the vector
\[
\langle a^3, a^2 - a, e^a \rangle
\]
is parallel to the plane \( \pi \).
2. Consider the lines given by parametric equations

\begin{align*}
L_1 : & \begin{cases} 
  x = t + 1 \\
  y = 3 + 2t \\
  z = 1 - t
\end{cases} \\
L_2 : & \begin{cases} 
  x = -s \\
  y = s + 1 \\
  z = 4 + s
\end{cases}
\end{align*}

a) (2 marks) Write down a direction vector for each of the two lines.

b) (2 marks) Show that the two lines are skew, i.e. they are not parallel and they do not intersect.

c) (2 marks) There is a unique plane \( \pi \) that contains \( L_1 \) and does not intersect \( L_2 \). Find an equation of the plane \( \pi \) (a sketch of any two skew lines in space might help to find a normal vector for the plane).

d) (2 marks) The distance between the two lines coincides with the distance between the plane \( \pi \) and any point on the line \( L_2 \). Find this distance.
3.

a) (2 marks) Describe and sketch the surface in space given by the equation $x = e^y$, and write down the coordinates of one point on the surface with $x$-coordinate equal to 5.

b) (2 marks) Given the quadric surface with equation $x^2 + y^2 + z = 5$, find the equation of one plane whose intersection with the surface is a circle of radius 1.

c) (2 marks) Sketch the quadric surface of point b).
[7] 4. Consider the function \( f(x, y) = \ln(x^2 + y - 4) \).

a) (3 marks) Determine and sketch the domain of \( f \).

b) (2 marks) Determine and sketch the \( k \)-level curve of \( f(x, y) \) for \( k = \ln 4 \).

c) (2 marks) For what values of \( a \) does the point \((a, 1, \ln 100)\) lie on the graph of \( f(x, y) \)?
5.

a) (3 marks) Find the first partial derivatives $f_x$ and $f_y$ of the function $f(x, y) = x^2 ye^{x+y}$.

b) (3 marks) Find all second partial derivatives and check that $f_{xy} = f_{yx}$.

c) (3 marks) For what values for the constant $a$ does the 2 variable function $f(x, y) = e^{ax^2y}$ satisfy the partial differential equation $x^3 \cdot f_x = 4y \cdot f_{yy}$?