Area between the curves, volumes, and centroids

Example 1 (easier, goal: to see similarities)
Consider the region R which lies between the curves $y = 1 - x^2$ and $y = x - 2$, $-1 \leq x \leq 1$.

(a) Set up the expression that represents the area of $R$

(b) Suppose that the region $R$ is revolved about $y = -3$. Set up the expression that represents the volume of the obtained solid.

© What is $x$-coordinate of the centroid of the region $R$?
Example 2 \( \int f(x) \, dx \), revolving about horizontal axis for intersecting curves.

Consider the region \( R \) which is bounded by

\[ y = x^2, \quad y = x \quad \text{for} \quad -2 \leq x \leq 2 \]

(a) Area of \( R \) (just set up)
(b) What is the volume of the solid obtained by revolving \( R \) about \( y = -2 \)? (set up only)
(c) \( \bar{x} \) - ? (just set up)
Example 3 (Integration with respect to $y$, revolving about vertical axis, $y$)

Consider the region $R$ bounded by all three curves $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, and $y = 4$.

(a) Area of $R$ (set up)

(b) Suppose that $R$ is revolved about $y = 0$. What is the volume of obtained solid? (setup)

(c) What is $y$-coordinate of the centroid of region $R$? (setup)
Example 4 Let's do same as in example 3 but for the region bounded by \( x = \frac{1}{y}, \quad x = \frac{1}{3y}, \quad \frac{1}{2} \leq y \leq 4 \).

Solution

(a) Find area.