Trigonometric substitutions. Part 2
(definite integrals containing $\sqrt{ax^2+bx+c}$)

Compute \( \int_{\frac{1}{2}}^{1} \frac{1}{(4x^2-4x+2)^2} \, dx \).

Solution: Completing the square, we get
\[
4x^2 - 4x + 2 = \left(2x - \frac{1}{2}\right)^2 + \frac{9}{4}.
\]
Let's rewrite the integral,
\[
\int_{\frac{1}{2}}^{1} \frac{1}{(4x^2-4x+2)^2} \, dx = \int_{\frac{1}{2}}^{1} \frac{1}{\left((2x - \frac{1}{2})^2 + \frac{9}{4}\right)^2} \, dx.
\]
Trigonometric substitutions, Part 2 (ctd)
Trigonometric substitution, Part 3

Example \( \int \frac{x}{\sqrt{1-4x^2}} \, dx \). (We will use the trigonometric substitution, but there is an easier way to compute the definite integral).

Solution
Example Compute \( \int \frac{dx}{(x^2 - 25)^{3/2}} \)

Solution
Integration of Rational Functions by Partial Fractions

Q: What is a rational function?
A: Rational function is

Examples

Task for today’s class: integrate rational functions.

Which rational functions from above can we integrate now? Circle them.
Partial Fractions with Simple Linear Factors

Goal: decompose the rational function into sum of rational functions which we can integrate.

Example: Decompose \( \frac{x^2 + x - 1}{x^2 - x} \)

Solution

Steps for \( \int \frac{N(x)}{P(x)} \, dx \)

Step 0: If degree of \( N(x) \) \( \geq \) degree of \( P(x) \), then do long division
Partial Fractions with Simple Linear Factors (etc)
Partial Fractions with Repeated Linear Factors

We have done the long division (step 0), and we factored the denominator (step 1), but we got repeated factors (linear ones).

Example: We are decomposing \( \frac{5x^2 - 3x + 2}{x^3 - 2x^2} \) and we got \( x^3 - 2x^2 = x \cdot x \cdot (x - 2) = x^2 (x - 2) \). What's next?

\[
\frac{5x^2 - 3x + 2}{x^3 - 2x^2} = \]

\[\text{repeated terms}\]
Partial Fractions with Irreducible Quadratic Factors

Some quadratic polynomials cannot be factored into linear terms.

Example Decompose \( \frac{7x^2 - 13x + 13}{(x-2)(x^2 - 2x + 3)} \).