Integration by parts
Before we used the chain rule for derivatives to obtain the substitution rule for integrals. Now any use of the product rule for derivatives to evaluating integrals?
Recall the product rule ($u(x)$ and $v(x)$ are differentiable)
$$[u(x)v(x)]' = u'(x)v(x) + u(x)v'(x).$$
Getting back to integrals Integrate both sides.
Integration by parts

Theorem (integration by parts)
Let \( u(x) \) and \( v(x) \) be continuously differentiable, i.e., \( u'(x) \) and \( v'(x) \) exist and are continuous. Then

\[
\int u(x)v'(x) \, dx = u(x)v(x) - \int v(x)u'(x) \, dx
\]

and

\[
\int_{a}^{b} u(x)v'(x) \, dx =
\]

The application of these formulas is known as integration by parts

Example: Compute \( \int x \log x \, dx \)
Solution: \( \int x \log x \, dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + C \)
Integration by parts

Example Compute $\int x \cos x \, dx$
Integration by parts

Example: Compute \( \int_0^{\pi/4} x^2 \sin x \, dx \)
Integration by parts

Example \( \int \ln x \, dx \)
Example: Let $R$ be the region bounded by $y = \ln x$, the $x$-axis, and the line $x = e$. Find the volume of the solid that is generated when the region $R$ is revolved about the $x$-axis.
Integration by parts. Special trick
(works for $\int e^x \sin x \, dx$ and $\int e^x \cos x \, dx$)

Example Compute $\int e^x \sin(2x) \, dx$
Trigonometric integrals

We study two types of integrals:

\[ \int \sin^m x \cos^n x \, dx \quad \text{and} \quad \int \tan^m x \sec^n x \, dx \]

We are done on Thursday.

We know that for \( \int \tan^m x \sec^n x \, dx \), if \( m \) is odd, use

\[ \tan x = \frac{\sin x}{\cos x} \quad \text{and} \quad \sec x = \frac{1}{\cos x} \]

Example: \( \int \tan x \sec^{2013} x \, dx \)
Trigonometric integrals $\int \tan^m x \sec^n x \, dx$

General approach

Is $m$ odd?

- yes
- no

Is $n$ odd?

- yes
  - $m$ is even, $n$ is odd
  - Beyond the scope of MATH 101
- no

$m$ is even, $n$ is even

Is $n \geq 2$?

- yes
  - $m$ is even, $n$ is even, $n \geq 2$
- no
  - $m$ is even, $n = 0$
Case In $\int \tan^m x \sec^n x \, dx$, $m$ is even, $n$ is even, $n \geq 2$.

\[ n = 2k + 2 \]

\[ \int \tan^m x \sec^{2k+2} x \, dx = \int \tan^m x (\sec^2 x)^k \sec^2 x \, dx \]

Example \[ \int \tan^2 x \sec^2 x \, dx \]
Solution \[ \int \tan^2 x \sec^2 x \, dx = \frac{1}{2} \sec^2 x \] let $u = \tan x$

Example \[ \int \tan^4 x \sec^4 x \, dx \]
Solution \[ \int \tan^4 x \sec^4 x \, dx = \int \tan^4 x \sec^2 x \sec^2 x \, dx \]
Case \( m \) is even, \( n = 0 \).

\[
m = 2k.
\]

\[
\int \tan^{2k} x \, dx
\]

Example \( \int \tan^2 x \, dx \).

Solution Using \( \tan^2 x = 1 - \sec^2 x \),

\[
\int \tan^2 x \, dx =
\]

Example \( \int \tan^4 x \, dx \).

Solution Using \( \tan^2 x = 1 - \sec^2 x \),

\[
\int \tan^4 x \, dx = \int \tan^2 x \cdot \tan^2 x \, dx =
\]