1. Chapter 2, Exercise 10, 13, 15

2. Let $A, B$ be disjoint subsets of $\mathbb{C}$ such that $A$ is compact and $B$ is closed. Show that there exists an $\epsilon > 0$ such that

$$|a - b| \geq \epsilon,$$

for all $a \in A, b \in B$. (Hint: show that the function $d_B : A \to \mathbb{R}$ defined by $d_B(a) = \inf_{b \in B} |a - b|$ is strictly positive and continuous.)

3. Let $u : \mathbb{C} \to \mathbb{R}$ be a bounded harmonic function that is twice continuously differentiable; that is, we identify $\mathbb{C}$ with $\mathbb{R}^2$ and require that $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \equiv 0$. Show that $u$ is constant. (Hint: Use a similar strategy as Exercise 12(a) of HW2 to construct a holomorphic function $f : \mathbb{C} \to \mathbb{C}$ such that $\text{Re}(f) = u$. What happens if you exponentiate $f$?)

Practice Problems (do not hand in)

1. Complete the proof of Corollary 4.2 by showing uniform convergence of the integral as $h$ tends to zero. (One option is to show uniform convergence of difference quotients as mentioned in the class.

Here is another method that avoids uniform convergence but uses the following version of Dominated convergence theorem instead. If $f_n : [a, b] \to \mathbb{C}$ is a sequence of functions that converge pointwise to $f : [a, b] \to \mathbb{C}$ (that is $\lim_{n \to \infty} f_n(t) = f(t)$ for all $t \in [a, b]$) and there exists $M > 0$ such that $|f_n(t)| \leq M$ for all $t \in [a, b]$ and all $n \geq 1$, then $\int_a^b f(t) \, dt = \lim_{n \to \infty} \int_a^b f_n(t) \, dt$.)

2. Exercise 1, 2, 3, 14