1. Let $R$ denote the radius of convergence of the power series $\sum_{k=0}^{\infty} a_k z^k$. Assume that $R < \infty$. For any $z \in \mathbb{C}$ with $|z| > R$, show that there is a sequence of $n_k \to \infty$ as $k \to \infty$ such that the partial sums $|\sum_{i=0}^{n_k} a_i z^i| \to \infty$ as $k \to \infty$. In other words, show that for any $M > 0$, there exists $K$ such that $|\sum_{i=0}^{n_k} a_i z^i| \geq M$ for all $k \geq K$.

2. Let $D$ be an open disc whose boundary circle is $C$, and let $z_0 \in D$. Using the Cauchy integral formula, show that there is a constant $K$ such that

$$|f(z_0)| \leq K \sup_{\zeta \in C} |f(\zeta)|$$

for any function $f : D \cup C \to \mathbb{C}$ that is continuous in $D \cup C$ and holomorphic in $D$. By considering $f(z)^n$ and letting $n \to \infty$ argue that the above estimate holds with $K = 1$. (Remark: This is the Maximum modulus principle. We will see an alternate proof in Theorem 4.5 of Chapter 3).

- (To be handed in) Chapter 2: Exercise 7 (verifying the equality case is not necessary), 9, 12 (You may assume the result of Exercise 11 without proof).

Practice problems (do not hand in)

- Chapter 2: Exercise 8, 10, Problem 1.