MATH 440/508
HW 1

Due September 19, 2018 (in class)

• (To be handed in) Chapter 1: Exercise 7, 9, 19(c), 22, 23.

• Practice problems (not to be handed in): Chapter 1: Exercise 5, 8, 13, 16, 19(a,b), 26

The practice problems below are meant to help review MATH 320 (do not hand in).

1. Let $\Omega \subset \mathbb{C}$ be a closed set. Let $z = \lim_{n \to \infty} z_n$, where $z_n \in \Omega$ for all $n \in \mathbb{N}$. Show that $z \in \Omega$.

2. Show that every convergent sequence in $\mathbb{C}$ is a Cauchy sequence.

3. Let $(a_n)$ be a bounded sequence of real numbers. Show that $a = \limsup_{n \to \infty} a_n$ can be characterized as the unique real number satisfying the two properties:
   
   (i) If $\alpha < a$, then there are infinitely many $n \in \mathbb{N}$ with $a_n \geq \alpha$.
   (ii) If $\beta > a$, then there are only finitely many $n \in \mathbb{N}$ such that $a_n \geq \beta$.

   Recall that $\limsup_{n \to \infty} a_n$ is defined as $\lim_{n \to \infty} b_n$, where $b_n = \sup_{m \geq n} a_n$.

4. Let $a_n$ be a sequence of complex numbers and let $s_n = \sum_{k=0}^{n} a_k$ denote the partial sums. If $\lim_{n \to \infty} s_n$ exists, then show that $\lim_{n \to \infty} a_n = 0$. 