January 12, 2018

MATH 318 Assignment 2: Due Friday, Jan 19 at start of class

I. Problems to be handed in:

1. A person's eye colour is determined by a pair of genes, one of which is inherited independently from each parent. Each gene can be blue-eye or brown-eye. A gene inherited from a parent is equally likely to be either of that parent's two genes. The brown-eye gene is *dominant* in the sense that if at least one gene is the brown-eye gene then eye colour will be brown, whereas if both genes are blue-eye genes then eye colour will be blue. Smith and both his parents have brown eyes, whereas Smith's sister has blue eyes.
   (a) What is the probability that Smith possesses the blue-eye gene?
   (b) Smith's wife has blue eyes. What is the probability that their first child will have blue eyes?
   (c) If their first child has brown eyes, what is the probability that their second child will also have brown eyes?

2. A true-false question is posed to a team with two members. Each team member independently gives the correct answer with probability p.
   (a) Which of the following is the better strategy for the team?
      i. Choose one team member at random and let that member answer the question.
      ii. Have each member decide his or her answer. If the answers agree, that is the team's answer. If they disagree, they flip a fair coin and answer "true" if the outcome is heads, and answer "false" if the outcome is tails.
   (b) Suppose $p = 0.6$ and the team adopts strategy ii. What is the conditional probability that the team gives the correct answer given that the team members disagree.

3. (a) Let E, F, G be independent events with probabilities $P(E) = 0.2, P(F) = 0.4$, and $P(G) = 0.7$. What is $P(E \cup F \cup G^c)$?
   (b) Let K, L, M be three events such that
      i. K and L are disjoint,
      ii. K and M are independent,
      iii. L and M are independent.
   Are $K \cup L$ and M independent events? Justify your answer.
   (c) Let A, B, C be independent events. Are $A \cup B$ and C independent? Justify your answer.

4. Two teams play a series of games, each of which is won by Team A with probability p and by Team B with probability $1 - p$. The winner of the series is the first team to win 4 games. Assume that the outcome of each game is independent. Find
   (a) Probability that a total of 6 games are played.
   (b) Conditional probability that Team A wins the third game given that Team A won the series and six games were played.
   (c) Conditional probability that a total of 6 games are played given that Team A won the first two games.
   Hint: For 4(c), one method is to use the answer to question 3(b).

5. (a) In Octave, simulate 100,000 geometric random variables with parameter $p = 0.01$ and create a histogram of the resulting values, with buckets for each of the values 1 to 1000. The octave command `geornd` should be useful; to learn more about it, use the `help` command by typing `help geornd`. (Note that in Octave, geometric random variables have values starting from 0, as they model the number of unsuccessful trials of an experiment *before* the first success; to convert these to geometric random variables as discussed in class, simply add one.)
   (b) Next, create a plot of the probability mass function of the geometric random variable, over the integers 1 to 1000. Here, you may use the Octave command `geopdf` (try `help geopdf`; in Octave the p.m.f. is referred to as "probability density function" or p.d.f., which we use later to mean something different). Again, since Octave considers geometric random variables to have values starting from 0, you will have to compensate for this by using `geopdf` to calculate the p.m.f. at the values 0 to 999. Briefly describe how this plot compares to the histogram from part (a).
   (c) Finally, plot the function $f(t) = e^{-t}$ for t between 0 and 10. Briefly, compare this plot to the two plots above. [1]

   Print and submit your program and output. Your code should be easy to read and adequately commented. Submit hard copies, no email, and do not use other programming languages.

II. Recommended problems: These provide additional practice but are not to be handed in. Starred problems have solutions in text, and answers are given otherwise.
Ross Chapter 1: 12, 13 (0.4929; one way is to use #11,12), 19*, 20 ($\frac{5}{12}$), 23, 25*, 30*, 33(9), 40*.
Chapter 2: 1 ($\{rr, oo, bb, ro, ob, br\}$; $\{0, 1, 2\}$; $\frac{7}{15}$), 4*.

[1] An explanation of the relation between plots in (b) and (c) will appear in Assignment 3.