1. Suppose $X, Y$ are two discrete RV’s with joint p.m.f. according to the table below.
   (a) Calculate the marginal p.m.f. of $X$ and of $Y$.
   (b) Calculate $\mathbb{P}(X^2 + Y < 3)$.
   (c) Are $X$ and $Y$ independent?

   \begin{table}[h]
   \centering
   \begin{tabular}{|c|c|c|c|c|}
   \hline
   $X \downarrow Y$ & 0 & 1 & 2 & 3 \\
   \hline
   1/2 & 1/12 & 1/8 & 1/8 & 1/12 \\
   1 & 0 & 1/12 & 1/9 & 1/9 \\
   6 & 1/12 & 1/12 & 0 & 1/9 \\
   \hline
   \end{tabular}
   \caption{The joint p.m.f. of $X, Y$}
   \end{table}

2. You have two dice, one with three sides labeled 0,1,2 and one with 4 sides, labeled 0,1,2,3. Let $X_1$ be the outcome of rolling the first die, and $X_2$ the outcome of rolling the second. The rolls are independent.
   (a) What is the joint p.m.f. of $(X_1, X_2)$
   (b) Let $Y_1 = X_1 \cdot X_2$ and $Y_2 = \max\{X_1, X_2\}$. Make a table for the joint p.m.f. of $(Y_1, Y_2)$.
   (c) Are $Y_1, Y_2$ independent?

3. Let $X \sim \text{Exp}(1/2)$, $Y \sim \text{Unif}([2,4])$, and assume that $X$ and $Y$ are independent.
   Calculate $\mathbb{P}(Y - X \geq \frac{1}{2})$.

4. Suppose that $X_1, \ldots, X_n$ are independent continuous random variables that all have the same c.d.f. $F(x)$. Define the random variable
   \[ Y = \max\{X_1, \ldots, X_n\}. \]
   Compute the c.d.f. and the p.d.f. of $Y$. Your answer should be in terms of $F(x)$.
   \textit{Hint:} Express an inequality of the kind $\max\{X_1, \ldots, X_n\} \leq b$ in terms of separate inequalities for each $X_i$.

5. The random variables $X, Y$ have joint probability density function
   \[ f(x, y) = \begin{cases} 
   Cy e^{-y-x/y} & \text{if } x > 0 \text{ and } y > 0, \\
   0 & \text{otherwise}. 
   \end{cases} \]
   (a) What is the value of $C$? \textit{Hint:} Integrate with respect to $x$ first.
   (b) Find the marginal probability density function $f_Y$.
   (c) Compute $\mathbb{P}(X \leq Y^2)$.
   (d)* Compute $\mathbb{P}(X \leq Y^3)$. Your result should be exact, and in terms of the c.d.f. of the standard normal.

6*. Let $Z_1$ and $Z_2$ be two points chosen uniformly from the unit disk, independently of each other. Let $d(Z_1, Z_2)$ denote their Euclidean distance, that is, if $z_i = (x_i, y_i)$, then
   \[ d(z_1, z_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}. \]
   Compute $\mathbb{E}(d(Z_1, Z_2)^2)$. 