1. Copying 1 billion \(10^9\) bits from a USB drive to your hard disk takes about 0.2 seconds under good conditions. However, every bit has a probability of roughly \(p = 10^{-8}\) to be copied incorrectly. Using error correcting codes, your PC is able to recognize and correct flawed bits during the copying process. Assume that it takes 0.02 seconds to correct a flawed bit. Use the Poisson approximation to calculate the probability that copying a movie of 2.5 gigabytes takes less than 25 seconds.

2. Let \(X\) be a Poisson random variable with parameter \(\lambda\)
   a) Which \(n = n(\lambda) \geq 0\) is the most likely value of \(X\), i.e. maximizes \(P(X = n)\)?
   b) Suppose the experiment described by \(X\) has returned the value \(n \geq 0\). Which parameter \(\lambda = \lambda(n)\) maximizes \(P(X = n)\)?

3. Suppose that the continuous RV \(X\) has c.d.f. given by
   
   \[
   F(x) = \begin{cases} 
   0 & \text{if } x < \frac{1}{\sqrt{2}} \\
   5 - 12\sqrt{2}x + 18x^2 - 4\sqrt{2}x^3 & \text{if } \frac{1}{\sqrt{2}} \leq x < \sqrt{2} \\
   1 & \text{if } \sqrt{2} \leq x
   \end{cases}
   \]

   (a) Find the smallest interval \([a, b]\) such that \(P(a \leq X \leq b) = 1\).
   (b) Find \(P(0 < X < \frac{1}{2})\).
   (c) Find \(P(X = 1)\).
   (d) Find \(P(1 \leq X \leq \frac{3}{2})\).
   (e) Find the p.d.f. of \(X\).

4. Define the function
   
   \[
   f(x) = \begin{cases} 
   9x^2 - 4x^3 + b & x \in [0, 1] \\
   0 & \text{otherwise}
   \end{cases}
   \]

   Show that there is no value of \(b\) for which this is the p.d.f. of some RV \(X\).

5. Suppose a continuous RV \(X\) has the p.d.f.
   
   \[
   f(x) = \begin{cases} 
   \frac{x}{1+x^2} & x > 0 \\
   0 & x \leq 0
   \end{cases}
   \]

   (a) Find the c.d.f. of \(X\).
   (b) What must be the value of \(c\)?
   (c) Find \(E(X)\).
   (d) Compute \(E\left(\frac{1}{\sqrt{1+X^2}}\right)\).

6. Let \(c > 0\) and \(X \sim \text{Unif}[0, c]\). Show that the RV \(Y = c - X\) has the same c.d.f. and therefore also the same p.d.f. as \(X\).

7. (a) Suppose that the duration \(T\) (in hours) of your morning routine (breakfast, shower, etc.) is modeled by an exponential RV with parameter \(\lambda\). You set your alarm 1 hour before your bus leaves for UBC. For which value of \(\lambda\) do you have a 50% chance of catching the bus?
   (b) Calculate the \(n\)th moment of \(T\).