Math 601D: Entropy in Information Theory, Ergodic Theory and Dynamical Systems

REVISED OUTLINE

W11-12 (Term 1: September - December, 2011)
MWF 11:00, Math Building 126

Instructor: Brian Marcus

marcus@math.ubc.ca
Math Bldg 218
(604) 822-3262

Office Hours: MWF 1:00

Overview: Entropy is a fundamental concept in all three subjects. In information theory, it represents the incompressibility of a probabilistic source and is used to quantify the maximal transmission rate possible over a noisy channel. In ergodic theory and topological dynamics it is a fundamental invariant associated to a dynamical system. It has been used successfully in classification problems. The three notions of entropy were developed in the 1940's, 1950's and 1960's respectively, and each has had a profound influence on the development of each subject ever since. The course will emphasize the connections among these different notions of entropy.

Topics:

1. Brief introduction to the origins of entropy in statistical mechanics
2. Shannon entropy for random variables and stationary stochastic processes (material taken from chapters 2,3,4 of [CT])
3. measure-theoretic entropy for measure-preserving transformations on finite measure spaces (material taken from chapters 1,2,4 of [W] and chapters 1,2,3 of [K])
4. topological entropy for continuous transformations of compact metric spaces, with emphasis on symbolic dynamical systems (material taken from chapters 5,7 of [W] and chapters 2,4 of [LM])

5. connections among topics 2,3 and 4: pressure, equilibrium states, variational principle, Gibbs states (material taken from chapters 6,8,9 of [W] and chapters 4,5 of [K])

Pre-requisites: Analysis, including measure theory, and probability theory, preferably at the graduate level. Please consult the instructor if you have any questions.

Target audience: Students in ergodic theory, probability theory, applied and pure harmonic analysis, and information and communication theory.

Evaluation: Each student will give a talk on a topic related to the course.

References: There is no required textbook. Material will be taken from the sources below. The main sources are [K] and W].

[CT] T. Cover and J. Thomas, Elements of Information Theory, Wiley Press, 2nd edition, 2006. (*an introduction to information theory*)

[K] G. Keller, Equilibrium States in Ergodic Theory, London Mathematical Society Student Notes, vol. 42, Cambridge U. Press, 1998 (*a more advanced introduction to ergodic theory with emphasis on greater generality and connections to ideas from statistical mechanics*)

[LM] D. Lind and B. Marcus, An Introduction to Symbolic Dynamics and Coding, Cambridge U. Press, 2nd printing, 1999 (*a concrete introduction to symbolic dynamics*)

[W] P. Walters, An Introduction to Ergodic Theory, Graduate Texts in Mathematics, v, 79, Springer-Verlag, 1982 (*a basic introduction to the fundamentals of ergodic theory*)