Overview: Entropy is a fundamental concept in all three subjects. In information theory, it represents the incompressibility of a probabilistic source and is used to quantify the maximal transmission rate possible over a noisy channel. In ergodic theory and topological dynamics it is a fundamental invariant associated to a dynamical system. It has been used successfully in classification problems. The three notions of entropy were developed in the 1940’s, 1950’s and 1960’s respectively, and each has had a profound influence on the development of each subject ever since. The course will emphasize the connections among these different notions of entropy.

Topics:

1. Brief introduction to the origins of entropy in statistical mechanics
2. Shannon entropy for random variables and stationary stochastic processes (material taken from chapters 2,3,4,6 of [CT])
3. measure-theoretic entropy for measure-preserving transformations on finite measure spaces (material taken from chapters 1,2,3 of [K])
4. topological entropy for continuous transformations of compact metric spaces, with emphasis on symbolic dynamical systems (material taken from chapters 2,4,6 of [LM])
5. connections among topics 2,3 and 4: pressure, equilibrium states, vari-
tional principle, Gibbs states (material taken from chapters 4,5 of [K])

Pre-requisites: Analysis, including measure theory, and probability the-
ory, preferably at the graduate level. Please consult the instructor if you
have any questions.

Target audience: Students in ergodic theory, probability theory, applied
and pure harmonic analysis, and information and communication theory.

References: Material will be taken from the following sources ([K] will be
the major source; if you purchase only one of these books, please choose [K]):

[CT] T. Cover and J. Thomas, Elements of Information Theory, Wiley

[K] G. Keller, Equilibrium States in Ergodic Theory, London Mathe-

[LM] D. Lind and B. Marcus, An Introduction to Symbolic Dynamics