JUSTIFY ALL OF YOUR ANSWERS. YOU MAY USE RESULTS FROM CLASS AND HOMEWORK.

CALCULATORS, NOTES OR BOOKS ARE NOT PERMITTED.

THERE ARE 4 PROBLEMS ON THIS EXAM.
1. Let $\vec{r}(t) = \left( (1/3)(1 + 2t)^{3/2}, (1/2)t^2 \right)$, $0 \leq t \leq 1$.

   (a) Find the arclength parameterization of the oriented curve which is the image of $\vec{r}(t)$.

   (b) Find the arclength parameterization of the same curve but with reversed orientation.
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2. Let \( \vec{r}(t) \) be a \( C^2 \) smooth parameterization, \( \vec{v}(t) := \vec{r}'(t) \), and \( \vec{a}(t) := \vec{r}''(t) \). Fix \( t \).

(a) Show that \( \vec{a}(t) \) is orthogonal to \( \vec{v}(t) \) iff \( ||\vec{v}(t)||' = 0 \).

(b) Show that \( \vec{a}(t) \) is parallel to \( \vec{v}(t) \) iff \( \kappa(t) = 0 \).
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Consider a $C^4$ smooth curve with nowhere-zero curvature. Let
\[ \overline{w}(s) = \tau(s)T(s) + \kappa(s)B(s) \]

(a) Show that $||\overline{w}(s)|| = \sqrt{(\kappa(s))^2 + (\tau(s))^2}$.

(b) Show that $\mathbf{N}'(s) = \overline{w}(s) \times \mathbf{N}(s)$

(c) Show that the curve is a helix iff $\overline{w}(s)$ is a constant vector.
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4. Let \( f(t) = t^4 \sin(1/t) \) if \( t \neq 0 \) and \( f(0) = 0 \). Let

\[
\vec{r}(t) = \begin{cases} 
(t, f(t), 0) & t \geq 0 \\
(t, 0, f(t)) & t < 0 
\end{cases}
\]

(a) Show that \( \vec{r}(t) \) is a smooth parameterization.

(b) Find the unit tangent vector at \( t = 0 \).

(c) Find the curvature at \( t = 0 \).

(d) Show that for each \( t \), either the curvature is 0 or the torsion is 0.