Math 226, HW5, due on Friday, October 20

1. Section 12.6: 1, 4, 17, 19

2. Chapter 12, Review Exercises: 4, 5

3. Let $a_1, \ldots, a_m$ be real numbers. Show that

$$|a_i| \leq \sqrt{\sum_{j=1}^{m} a_j^2} \leq \sum_{i=1}^{m} |a_i|$$

(the first inequality holds for all $i$).

4. Recall that for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$

$$|\mathbf{x} - \mathbf{y}| := \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Write $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ as $\mathbf{f}(x_1, \ldots, x_n) = (f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n))$. Let $\mathbf{L} = (L_1, \ldots, L_m) \in \mathbb{R}^m$.

Precise $\epsilon - \delta$ definition of $\lim_{x \to x_0} \mathbf{f}(x) = \mathbf{L}$:

$$\forall \epsilon > 0 \exists \delta > 0 \text{ such that if } 0 < |x - x_0| < \delta, \text{ then } |\mathbf{f}(x) - \mathbf{L}| < \epsilon.$$ 

Show that

$$\lim_{x \to x_0} f(x) = L \text{ iff for each } i = 1, \ldots, m, \lim_{x \to x_0} f_i(x) = L_i.$$