39. $f(x) = x^n$
$g(x) = -x^n = -f(x), \quad n = 2, 3, 4, \ldots$
$f_n'(x) = nx^{n-1} = 0$ at $x = 0$
If n is even, f_n has a loc min, g_n has a loc max at $x = 0$.
If n is odd, f_n has an inflection at $x = 0$, and so does g_n.

40. Let there be a function f such that

$$f'(x_0) = f''(x_0) = \ldots = f^{(k-1)}(x_0) = 0,$$
$$f^{(k)}(x_0) \neq 0 \qquad \text{for some } k \geq 2.$$

If k is even, then f has a local min value at $x = x_0$ when $f^{(k)}(x_0) > 0$, and f has a local max value at $x = x_0$ when $f^{(k)}(x_0) < 0$.
If k is odd, then f has an inflection point at $x = x_0$.

#2

41. $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

a) $\lim_{x\to 0+} x^{-n} f(x) = \lim_{x\to 0+} \dfrac{e^{-1/x^2}}{x^n}$ (put $y = 1/x$)
$= \lim_{y\to\infty} y^n e^{-y^2} = 0$ by Theorem 5 of Sec. 4.4
Similarly, $\lim_{x\to 0-} x^{-n} f(x) = 0$, and $\lim_{x\to 0} x^{-n} f(x) = 0$.

b) If $P(x) = \sum_{j=0}^{n} a_j x^j$ then by (a)

$$\lim_{x\to 0} P\left(\frac{1}{x}\right) f(x) = \sum_{j=0}^{n} a_j \lim_{x\to 0} x^{-j} f(x) = 0.$$

c) If $x \neq 0$ and $P_1(t) = 2t^3$, then

$$f'(x) = \frac{2}{x^3} e^{-1/x^2} = P_1\left(\frac{1}{x}\right) f(x).$$

Assume that $f^{(k)}(x) = P_k\left(\dfrac{1}{x}\right) f(x)$ for some $k \geq 1$, where P_k is a polynomial. Then

$$\begin{aligned} f^{(k+1)}(x) &= -\frac{1}{x^2} P_k'\left(\frac{1}{x}\right) f(x) + P_k\left(\frac{1}{x}\right) P_1\left(\frac{1}{x}\right) f(x) \\ &= P_{k+1}\left(\frac{1}{x}\right) f(x), \end{aligned}$$

where $P_{k+1}(t) = t^2 P_k'(t) + P_1(t) P_k(t)$ is a polynomial.
By induction, $f^{(n)} = P_n\left(\dfrac{1}{n}\right) f(x)$ for $n \neq 0$, where P_n is a polynomial.

d) $f'(0) = \lim_{h\to 0} \dfrac{f(h) - f(0)}{h} = \lim_{h\to 0} h^{-1} f(h) = 0$ by (a). Suppose that $f^{(k)}(0) = 0$ for some $k \geq 1$. Then

$$\begin{aligned} f^{(k+1)}(0) &= \lim_{h\to 0} \frac{f^{(k)}(h) - f^{(k)}(0)}{h} \\ &= \lim_{h\to 0} h^{-1} f^{(k)}(h) \\ &= \lim_{h\to 0} h^{-1} P_k\left(\frac{1}{h}\right) f(h) = 0 \end{aligned}$$

by (b).
Thus $f^{(n)}(0) = 0$ for $n = 1, 2, \ldots$ by induction.

e) Since $f'(x) < 0$ if $x < 0$ and $f'(x) > 0$ if $x > 0$, therefore f has a local min value at 0 and $-f$ has a loc max value there.

f) If $g(x) = xf(x)$ then $g'(x) = f(x) + xf'(x)$, $g''(x) = 2f'(x) + xf''(x)$.
In general, $g^{(n)}(x) = nf^{(n-1)}(x) + xf^{(n)}(x)$ (by induction).
Then $g^{(n)}(0) = 0$ for all n (by (d)).
Since $g(x) < 0$ if $x < 0$ and $g(x) > 0$ if $x > 0$, g cannot have a max or min value at 0. It must have an inflection point there.

42. We are given that

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$$

If $x \neq 0$, then

$$f'(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$
$$f''(x) = 2 \sin \frac{1}{x} - \frac{2}{x} \cos \frac{1}{x} - \frac{1}{x^2} \sin \frac{1}{x}.$$

If $x = 0$, then

$$f'(x) = \lim_{h\to 0} \frac{h^2 \sin \frac{1}{h} - 0}{h} = 0.$$

Thus 0 is a critical point of f. There are points x arbitrarily close to 0 where $f(x) > 0$, for example $x = \dfrac{2}{(4n+1)\pi}$, and other such points where $f(x) < 0$, for example $x = \dfrac{2}{(4n+3)\pi}$. Therefore f does not have a local max or min at $x = 0$. Also, there are points arbitrarily close to 0 where $f''(x) > 0$, for example $x = \dfrac{1}{(2n+1)\pi}$, and other such points where $f''(x) < 0$, for instance $x = \dfrac{1}{2n\pi}$. Therefore f does not have constant concavity on any interval $(0, a)$ where $a > 0$, so 0 is not an inflection point of f either.

27. $y = \dfrac{x^3 - 3x^2 + 1}{x^3} = 1 - \dfrac{3}{x} + \dfrac{1}{x^3}$

$y' = \dfrac{3}{x^2} - \dfrac{3}{x^4} = \dfrac{3(x^2-1)}{x^4}$

$y'' = -\dfrac{6}{x^3} + \dfrac{12}{x^5} = 6\dfrac{2-x^2}{x^5}$

From y : Asymptotes: $y = 1$, $x = 0$. Symmetry: none.
Intercepts: since $\lim_{x\to 0+} y = \infty$, and $\lim_{x\to 0-} y = -\infty$, there are intercepts between -1 and 0, between 0 and 1, and between 2 and 3.
Points: $(-1, 3)$, $(1, -1)$, $(2, -\frac{3}{8})$, $(3, \dfrac{1}{27})$.
From y': CP: $x = \pm 1$.

		CP		ASY		CP	
y'	+	-1	$-$	0	$-$	1	+
y	$\nearrow$	loc max	$\searrow$		$\searrow$	loc min	$\nearrow$

From y'': $y'' = 0$ at $x = \pm\sqrt{2}$.

				ASY			
y''	+	$-\sqrt{2}$	$-$	0	+	$\sqrt{2}$	$-$
y	$\smile$	infl	$\frown$		$\smile$	infl	$\frown$

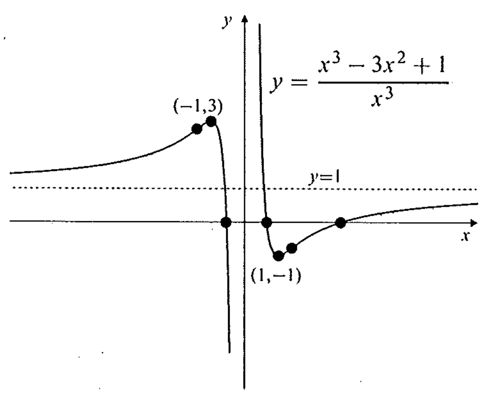

Fig. 4.6.27

28. $y = x + \sin x$, $y' = 1 + \cos x$, $y'' = -\sin x$.
#3
From y: Intercept: $(0, 0)$. Other points: $(k\pi, k\pi)$, where k is an integer. Symmetry: odd.
From y': Critical point: $x = (2k+1)\pi$, where k is an integer.

		CP		CP		CP	
f'	+	$-\pi$	+	π	$-$	3π	+
f	$\nearrow$		$\nearrow$		$\nearrow$		$\nearrow$

From y'': $y'' = 0$ at $x = k\pi$, where k is an integer.

y''	+	-2π	$-$	$-\pi$	+	0	$-$	π	+	2π	$-$
y	$\smile$	infl	$\frown$	infl	$\smile$	infl	$\frown$	infl	$\smile$	infl	$\frown$

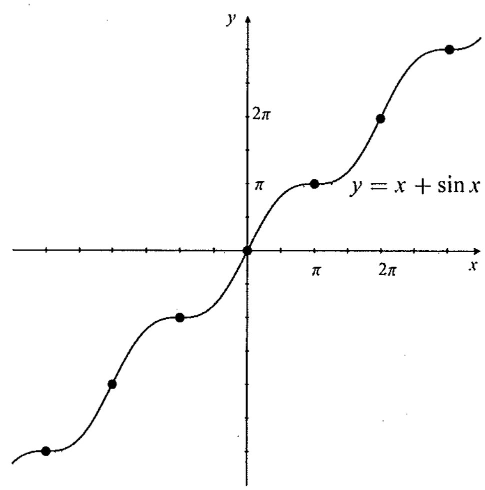

Fig. 4.6.28

Similar to #4

29. $y = x + 2\sin x$, $y' = 1 + 2\cos x$, $y'' = -2\sin x$.
$y = 0$ if $x = 0$
$y' = 0$ if $x = -\dfrac{1}{2}$, i.e., $x = \pm\dfrac{2\pi}{3} \pm 2n\pi$
$y'' = 0$ if $x = \pm n\pi$
From y: Asymptotes: (none). Symmetry: odd.
Points: $\left(\pm\dfrac{2\pi}{3}, \pm\dfrac{2\pi}{3} + \sqrt{3}\right)$, $\left(\pm\dfrac{8\pi}{3}, \pm\dfrac{8\pi}{3} + \sqrt{3}\right)$, $\left(\pm\dfrac{4\pi}{3}, \pm\dfrac{4\pi}{3} - \sqrt{3}\right)$.
From y': CP: $x = \pm\dfrac{2\pi}{3} \pm 2n\pi$.

		CP		CP		CP		CP		CP	
y'	$-$	$-\frac{8\pi}{3}$	+	$-\frac{4\pi}{3}$	+	$-\frac{2\pi}{3}$	+	$\frac{2\pi}{3}$	$-$	$\frac{4\pi}{3}$	+
y	$\searrow$	loc min	$\nearrow$	loc max	$\searrow$	loc min	$\nearrow$	loc max	$\searrow$	loc min	$\nearrow$

From y'': $y'' = 0$ at $x = \pm n\pi$.

y''	+	-2π	$-$	$-\pi$	+	0	$-$	π	+	2π	$-$
y	$\smile$	infl	$\frown$	infl	$\smile$	infl	$\frown$	infl	$\smile$	infl	$\frown$

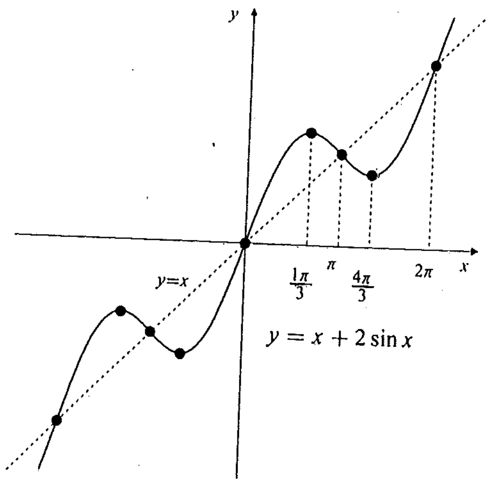

Fig. 4.6.29

30. $y = e^{-x^2}$, $y' = -2xe^{-x^2}$, $y'' = (4x^2 - 2)e^{-x^2}$.
From y: Intercept: $(0, 1)$. Asymptotes: $y = 0$ (horizontal). Symmetry: even.
From y': Critical point: $x = 0$.

#5

		CP 0	
y'	$+$		$-$
y	$\nearrow$	abs max	$\searrow$

From y'': $y'' = 0$ at $x = \pm\dfrac{1}{\sqrt{2}}$.

		$-\frac{1}{\sqrt{2}}$		$\frac{1}{\sqrt{2}}$	
y''	$+$		$-$		$+$
y	$\smile$	infl	$\frown$	infl	$\smile$

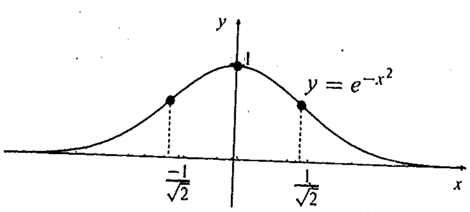

Fig. 4.6.30

31. $y = xe^x$, $y' = e^x(1 + x)$, $y'' = e^x(2 + x)$.
From y: Asymptotes: $y = 0$ (at $x = -\infty$).
Symmetry: none. Intercept $(0, 0)$.
Points: $\left(-1, -\dfrac{1}{e}\right)$, $\left(-2, -\dfrac{2}{e^2}\right)$,
From y': CP: $x = -1$.

		CP -1	
y'	$-$		$+$
y	$\searrow$	abs min	$\nearrow$

From y'': $y'' = 0$ at $x = -2$.

		-2	
y''	$-$		$+$
y	$\frown$	infl	$\smile$

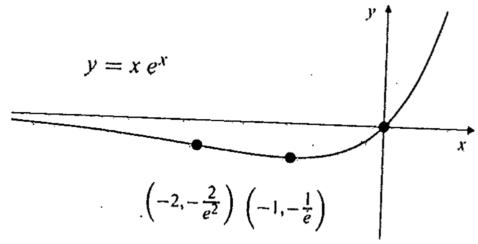

Fig. 4.6.31

32. $y = e^{-x}\sin x \quad (x \ge 0)$,
$y' = e^{-x}(\cos x - \sin x)$, $y'' = -2e^{-x}\cos x$.
From y: Intercept: $(k\pi, 0)$, where k is an integer.
Asymptotes: $y = 0$ as $x \to \infty$.
From y': Critical points: $x = \dfrac{\pi}{4} + k\pi$, where k is an integer.

	0		CP $\frac{\pi}{4}$		CP $\frac{5\pi}{4}$		CP $\frac{9\pi}{4}$	
y'		$+$		$-$		$+$		$-$
y		$\nearrow$	abs max	$\searrow$	abs min	$\nearrow$	loc max	$\searrow$

From y'': $y'' = 0$ at $x = (k + \frac{1}{2})\pi$, where k is an integer.

	0		$\frac{\pi}{2}$		$\frac{3\pi}{2}$		$\frac{5\pi}{2}$	
y''		$-$		$+$		$-$		$+$
y		$\frown$	infl	$\smile$	infl	$\frown$	infl	$\smile$

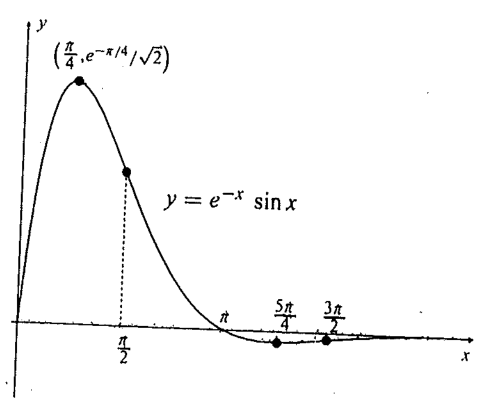

Fig. 4.6.32

Since $V(0) = V(35) = 0$, the maximum value will occur at a critical point:

$$\begin{aligned} 0 = V'(x) &= 4(2625 - 220x + 3x^2) \\ &= 4(3x - 175)(x - 15) \\ \Rightarrow x &= 15 \text{ or } \frac{175}{3}. \end{aligned}$$

The only critical point in $[0, 35]$ is $x = 15$. Thus, the largest possible volume for the box is

$$V(15) = 15(70 - 30)(150 - 30) = 72,000\,\text{cm}^3.$$

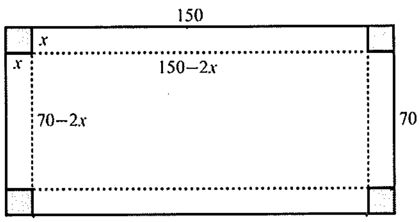

Fig. 4.8.18

19. Let the rebate be $\$x$. Then number of cars sold per month is

#6

$$2000 + 200\left(\frac{x}{50}\right) = 2000 + 4x.$$

The profit per car is $1000 - x$, so the total monthly profit is

$$\begin{aligned} P &= (2000 + 4x)(1000 - x) = 4(500 + x)(1000 - x) \\ &= 4(500,000 + 500x - x^2). \end{aligned}$$

For maximum profit:

$$0 = \frac{dP}{dx} = 4(500 - 2x) \Rightarrow x = 250.$$

(Since $\dfrac{d^2P}{dx^2} = -8 < 0$ any critical point gives a local max.) The manufacturer should offer a rebate of \$250 to maximize profit.

20. If the manager charges $\$(40+x)$ per room, then $(80-2x)$ rooms will be rented.
The total income will be $\$(80 - 2x)(40 + x)$ and the total cost will be $\$(80 - 2x)(10) + (2x)(2)$. Therefore, the profit is

$$\begin{aligned} P(x) &= (80 - 2x)(40 + x) - [(80 - 2x)(10) + (2x)(2)] \\ &= 2400 + 16x - 2x^2 \qquad \text{for } x > 0. \end{aligned}$$

If $P'(x) = 16 - 4x = 0$, then $x = 4$. Since $P''(x) = -4 < 0$, P must have a maximum value at $x = 4$. Therefore, the manager should charge \$44 per room.

21. Head for point C on road x km east of A. Travel time is

$$T = \frac{\sqrt{12^2 + x^2}}{15} + \frac{10 - x}{39}.$$

We have
$$\begin{aligned} T(0) &= \frac{12}{15} + \frac{10}{39} = 1.0564 \text{ hrs} \\ T(10) &= \frac{\sqrt{244}}{15} = 1.0414 \text{ hrs} \end{aligned}$$
For critical points:

$$\begin{aligned} 0 = \frac{dT}{dx} &= \frac{1}{15}\frac{x}{\sqrt{12^2 + x^2}} - \frac{1}{39} \\ &\Rightarrow 13x = 5\sqrt{12^2 + x^2} \\ &\Rightarrow (13^2 - 5^2)x^2 = 5^2 \times 12^2 \Rightarrow x = 5 \end{aligned}$$

$$T(5) = \frac{13}{15} + \frac{5}{39} = 0.9949 < \begin{cases} T(0) \\ T(10). \end{cases}$$
(Or note that

$$\begin{aligned} \frac{d^2T}{dt^2} &= \frac{1}{15}\frac{\sqrt{12^2 + x^2} - \dfrac{x^2}{\sqrt{12^2 + x^2}}}{12^2 + x^2} \\ &= \frac{12^2}{15(12^2 + x^2)^{3/2}} > 0 \end{aligned}$$

so any critical point is a local minimum.)
To minimize travel time, head for point 5 km east of A.

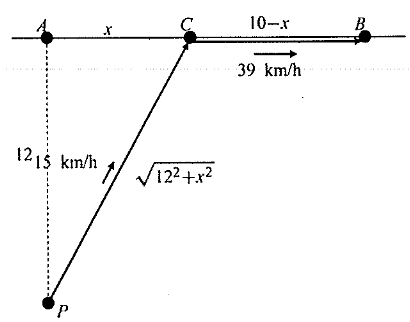

Fig. 4.8.21

22. This problem is similar to the previous one except that the 10 in the numerator of the second fraction in the expression for T is replaced with a 4. This has no effect on the critical point of T, namely $x = 5$, which now lies outside the appropriate interval $0 \le x \le 4$. Minimum T must occur at an endpoint. Note that

$$\begin{aligned} T(0) &= \frac{12}{15} + \frac{4}{39} = 0.9026 \\ T(4) &= \frac{1}{15}\sqrt{12^2 + 4^2} = 0.8433. \end{aligned}$$

#7

30. Let θ be the angle of inclination of the ladder. The height of the fence is

$$h(\theta) = 6\sin\theta - 2\tan\theta \qquad \left(0 < \theta < \frac{\pi}{2}\right).$$

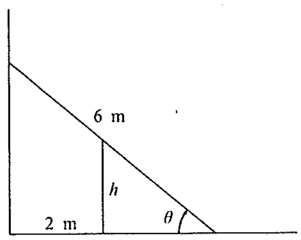

Fig. 4.8.30

For critical points:

$$\begin{aligned} 0 &= h'(\theta) = 6\cos\theta - 2\sec^2\theta \\ &\Rightarrow 3\cos\theta = \sec^2\theta \Rightarrow 3\cos^3\theta = 1 \\ &\Rightarrow \cos\theta = \left(\tfrac{1}{3}\right)^{1/3}. \end{aligned}$$

Since $h''(\theta) = -6\sin\theta - 4\sec^2\theta\tan\theta < 0$ for $0 < \theta < \dfrac{\pi}{2}$, therefore $h(\theta)$ must be maximum at $\theta = \cos^{-1}\left(\frac{1}{3}\right)^{1/3}$. Then

$$\sin\theta = \frac{\sqrt{3^{2/3}-1}}{3^{1/3}}, \qquad \tan\theta = \sqrt{3^{2/3}-1}.$$

Thus, the maximum height of the fence is

$$\begin{aligned} h(\theta) &= 6\left(\frac{\sqrt{3^{2/3}-1}}{3^{1/3}}\right) - 2\sqrt{3^{2/3}-1} \\ &= 2(3^{2/3}-1)^{3/2} \approx 2.24\text{ m}. \end{aligned}$$

31. Let (x, y) be a point on $x^2y^4 = 1$. Then $x^2y^4 = 1$ and the square of distance from (x, y) to $(0, 0)$ is
$S = x^2 + y^2 = \dfrac{1}{y^4} + y^2, \quad (y \neq 0)$
Clearly, $S \to \infty$ as $y \to 0$ or $y \to \pm\infty$, so minimum S must occur at a critical point. For CP:

$$0 = \frac{dS}{dy} = \frac{-4}{y^5} + 2y \Rightarrow y^6 = 2 \Rightarrow y = \pm 2^{1/6}$$
$$\Rightarrow x = \pm\frac{1}{2^{1/3}}$$

Thus the shortest distance from origin to curve is

$$S = \sqrt{\frac{1}{2^{2/3}} + 2^{1/3}} = \sqrt{\frac{3}{2^{2/3}}} = \frac{3^{1/2}}{2^{1/3}}\text{ units.}$$

32. The square of the distance from $(8, 1)$ to the curve $y = 1 + x^{3/2}$ is

$$\begin{aligned} S &= (x-8)^2 + (y-1)^2 \\ &= (x-8)^2 + (1 + x^{3/2} - 1)^2 \\ &= x^3 + x^2 - 16x + 64. \end{aligned}$$

Note that y, and therefore also S, is only defined for $x \geq 0$. If $x = 0$ then $S = 64$. Also, $S \to \infty$ if $x \to \infty$. For critical points:

$$\begin{aligned} 0 = \frac{dS}{dx} &= 3x^2 + 2x - 16 = (3x+8)(x-2) \\ &\Rightarrow x = -\tfrac{8}{3} \text{ or } 2. \end{aligned}$$

Only $x = 2$ is feasible. At $x = 2$ we have $S = 44 < 64$. Therefore the minimum distance is $\sqrt{44} = 2\sqrt{11}$ units.

33. Let the cylinder have radius r and height h. By symmetry, the centre of the cylinder is at the centre of the sphere. Thus

$$r^2 + \frac{h^2}{4} = R^2.$$

The volume of cylinder is

$$V = \pi r^2 h = \pi h\left(R^2 - \frac{h^2}{4}\right), \qquad (0 \leq h \leq 2R).$$

Clearly, $V = 0$ if $h = 0$ or $h = 2R$, so maximum V occurs at a critical point. For CP:

$$\begin{aligned} 0 = \frac{dV}{dh} &= \pi\left[R^2 - \frac{h^2}{4} - \frac{2h^2}{4}\right] \\ \Rightarrow h^2 &= \frac{4}{3}R^2 \qquad \Rightarrow h = \frac{2R}{\sqrt{3}} \\ \Rightarrow r &= \sqrt{\frac{2}{3}}R. \end{aligned}$$

The largest cylinder has height $\dfrac{2R}{\sqrt{3}}$ units and radius $\sqrt{\dfrac{2}{3}}R$ units.

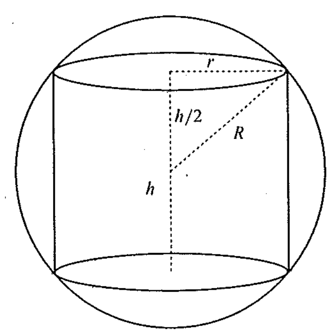

Fig. 4.8.33

$f''(x) < 0$ on $[81, 85]$ so error is negative: $\sqrt[4]{85} < \frac{82}{27}$.
$|f''(x)| < \frac{3}{16 \times 3^7} = \frac{1}{11,664} = k$ on $[81, 85]$.
Thus $|\text{Error}| \leq \frac{k}{2}(85 - 81)^2 = 0.00069$.

$$\frac{82}{27} - \frac{1}{1458} < \sqrt[4]{85} < \frac{82}{27},$$

or $3.036351 \leq \sqrt[4]{85} \leq 3.037037$

18. Let $f(x) = \frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$ and $f''(x) = \frac{2}{x^3}$. Hence,

$$\begin{aligned}\frac{1}{2.003} &= f(2.003) \approx f(2) + f'(2)(0.003)\\ &= \frac{1}{2} + \left(-\frac{1}{4}\right)(0.003) = 0.49925.\end{aligned}$$

If $x \geq 2$, then $|f''(x)| \leq \frac{2}{8} = \frac{1}{4}$. Since $f''(x) > 0$ for $x > 0$, f is concave up. Therefore, the error

$$E = \frac{1}{2.003} - 0.49925 > 0$$

and

$$|E| < \frac{1}{8}(0.003)^2 = 0.000001125.$$

Thus,

$$\begin{aligned}0.49925 &< \frac{1}{2.003} < 0.49925 + 0.000001125\\ 0.49925 &< \frac{1}{2.003} < 0.499251125.\end{aligned}$$

#8

19. $f(x) = \cos x, \quad f'(x) = -\sin x, \quad f''(x) = -\cos x$

$$\begin{aligned}\cos 46° &= \cos\left(\frac{\pi}{4} + \frac{\pi}{180}\right)\\ &\approx \cos\frac{\pi}{4} - \sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{180}\right)\\ &= \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{180}\right) \approx 0.694765.\end{aligned}$$

$f''(0) < 0$ on $[45°, 46°]$ so

$$|\text{Error}| < \frac{1}{2\sqrt{2}}\left(\frac{\pi}{180}\right)^2 \approx 0.0001.$$

We have

$$\frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{180} - \frac{\pi^2}{2 \times 180^2}\right) < \cos 46° < \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{180}\right)$$

i.e., $0.694658 \leq \cos 46° < 0.694765$.

20. Let $f(x) = \sin x$, then $f'(x) = \cos x$ and $f''(x) = -\sin x$. Hence,

$$\begin{aligned}\sin\left(\frac{\pi}{5}\right) &= f\left(\frac{\pi}{6} + \frac{\pi}{30}\right) \approx f\left(\frac{\pi}{6}\right) + f'\left(\frac{\pi}{6}\right)\left(\frac{\pi}{30}\right)\\ &= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{30}\right) \approx 0.5906900.\end{aligned}$$

If $x \leq \frac{\pi}{4}$, then $|f''(x)| \leq \frac{1}{\sqrt{2}}$. Since $f''(x) < 0$ on $0 < x \leq 90°$, f is concave down. Therefore, the error E is negative and

$$|E| < \frac{1}{2\sqrt{2}}\left(\frac{\pi}{30}\right)^2 = 0.0038772.$$

Thus,

$$\begin{aligned}0.5906900 - 0.0038772 &< \sin\left(\frac{\pi}{5}\right) < 0.5906900\\ 0.5868128 &< \sin\left(\frac{\pi}{5}\right) < 0.5906900.\end{aligned}$$

21. Let $f(x) = \sin x$, then $f'(x) = \cos x$ and $f''(x) = -\sin x$. The linearization at $x = \pi$ gives:

$$\sin(3.14) \approx \sin\pi + \cos\pi(3.14 - \pi) = \pi - 3.14 \approx 0.001592654.$$

Since $f''(x) < 0$ between 3.14 and π, the error E in the above approximation is negative: $\sin(3.14) < 0.001592654$. For $3.14 \leq t \leq \pi$, we have

$$|f''(t)| = \sin t \leq \sin(3.14) < 0.001592654.$$

Thus the error satisfies

$$|E| \leq \frac{0.001592654}{2}(3.14 - \pi)^2 < 0.000000002.$$

Therefore $0.001592652 < \sin(3.14) < 0.001592654$.

22. Let $f(x) = \sin x$, then $f'(x) = \cos x$ and $f''(x) = -\sin x$. The linearization at $x = 30° = \pi/6$ gives

$$\begin{aligned}\sin(33°) &= \sin\left(\tfrac{\pi}{6} + \tfrac{\pi}{60}\right)\\ &\approx \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\left(\frac{\pi}{60}\right)\\ &= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{60}\right) \approx 0.545345.\end{aligned}$$

Since $f''(x) < 0$ between 30° and 33°, the error E in the above approximation is negative: $\sin(33°) < 0.545345$. For $30° \leq t \leq 33°$, we have

$$|f''(t)| = \sin t \leq \sin(33°) < 0.545345.$$

Thus the error satisfies

$$|E| \leq \frac{0.545345}{2}\left(\frac{\pi}{60}\right)^2 < 0.000747.$$