1. Given a nonconstant non-decreasing function \( \alpha : [a, b] \to \mathbb{R} \), let \( \mathcal{R}_\alpha[a, b] \) denote the collection of all bounded functions on \([a, b]\) which are Riemann-Stieltjes integrable with respect to \(\alpha\). Is \(\mathcal{R}_\alpha[a, b]\) a vector space, a lattice, an algebra?

2. This problem focuses on computing the Riemann-Stieltjes integral for specific choices of integrators.
   (a) Let \(x_0 = a < x_1 < x_2 < \cdots < x_n = b\) be a fixed collection of points in \([a, b]\), and let \(\alpha\) be an increasing step function on \([a, b]\) that is constant on each of the open intervals \((x_{i-1}, x_i)\) and has jumps of size \(\alpha_i = \alpha(x_{i+}) - \alpha(x_i)\) at each of the points \(x_i\). For \(i = 0\) and \(n\), we make the obvious adjustments
      \[
      \alpha_0 = \alpha(a+) - \alpha(a), \quad \alpha_n = \alpha(b) - \alpha(b-).
      \]
   If \(f \in B[a, b]\) is continuous at each of the points \(x_i\), show that \(f \in \mathcal{R}_\alpha[a, b]\) and
   \[
   \int_a^b f \, d\alpha = \sum_{i=0}^n f(x_i)\alpha_i.
   \]
   (b) If \(f\) is continuous on \([1, n]\), compute \(\int_1^n f(x) d[x]\), where \([x]\) is the greatest integer in \(x\). What is the value of \(\int_1^t f(x) d[x]\) if \(t\) is not an integer?

3. Determine, with adequate justification, whether each of the following statements is true or false.
   (a) An equicontinuous, pointwise bounded subset of \(C[a, b]\) is compact.
   (b) The function \(\chi_Q\) is Riemann integrable on \([0, 1]\).
   (c) The function \(\chi_\Delta\) is Riemann integrable on \([0, 1]\), where \(\Delta\) denotes the Cantor middle-third set. (We have already run into this set in Homework 2, Problem 5).
   (d) \(\bigcap_\alpha \{ \mathcal{R}_\alpha[a, b] : \alpha\ \text{increasing} \} = C[a, b]\).
   (e) If \(f\) is a monotone function and \(\alpha\) is both continuous and non-decreasing, then \(f \in \mathcal{R}_\alpha[a, b]\).
   (f) There exists a non-decreasing function \(\alpha : [a, b] \to \mathbb{R}\) and a function \(f \in \mathcal{R}_\alpha[a, b]\) such that \(f\) and \(\alpha\) share a common-sided discontinuity.
   (g) If \(f \in \mathcal{R}_\alpha[a, b]\) with \(m \leq f \leq M\) and if \(\varphi\) is continuous on \([m, M]\), then \(\varphi \circ f \in \mathcal{R}_\alpha[a, b]\).