Two sets $A$ and $B$ are said to be equivalent if there exists a bijection $f : A \to B$, i.e., if $f$ is a one-to-one function from $A$ onto $B$.

A set $A$ is called finite if either $A = \emptyset$ or if $A$ is equivalent to the set $\{1, 2, \cdots, n\}$ for some $n$.

If $A$ is not finite, it is said to be infinite.

An infinite set is said to be countably infinite if it is equivalent to the set $\mathbb{N} = \{1, 2, 3, \cdots\}$ of natural numbers. Thus the elements of a countably infinite set can be enumerated or counted according to their correspondence with the natural numbers: $A = \{x_1, x_2, x_3, \cdots\}$ where the $x_i$ are distinct.

A set is countable if it is either finite or countably infinite.

Which of the following statements are true?

1. The set $[0, 1]$ is countable.

2. The countable union of countable sets is countable.

3. The set of rational numbers is countable.

4. The set of real numbers whose decimal expansions contain only 3 and 7 is countable.
A set $A$ is said to be dense in a metric space $(M, d)$ if $\overline{A} = \text{closure of } A = M$. More explicitly, $A$ is dense in $M$ if for every $x \in M$ and every $\epsilon > 0$, the following relation holds: $A \cap B(x; \epsilon) \neq \emptyset$. Thus, there exists a sequence $\{x_n\} \subseteq A$ such that $x_n \to x$.

A metric space is called separable if it contains a countable dense subset.

Which of the following statements are true?

1. Both $\mathbb{Q}$ and $\mathbb{R} \setminus \mathbb{Q}$ are dense in $\mathbb{R}$.

2. The space $\ell^2(\mathbb{R})$ of real square-summable sequences, namely

$$\ell^2(\mathbb{R}) = \{x = \{x_n : n \geq 1\} : \sum_{n=1}^{\infty} |x_n|^2 < \infty\}$$

is separable. Note that $\ell^2(\mathbb{R})$ is a metric space generated by the $\ell^2$ norm $\|x\|_2 = \sqrt{\sum_{n=1}^{\infty} |x_n|^2}$. 