1. Find the limit of the function \( f(z) = (z/\bar{z})^2 \), if it exists, as \( z \) tends to zero. If you think the limit does not exist, explain your reasoning for this conclusion.

   \( \text{(Answer: The limit does not exist.)} \)

2. Describe geometrically the collection of points \( z \) satisfying the equation \( |z - 1| = |z + i| \).
   Sketch this set of points in the complex plane.

   \( \text{(Answer: A straight line through the origin with slope } -1) \)

3. Express the complex number \((-1 + i)^7\) in the form \( a + ib \).

   \( \text{(Answer: } -8(1 + i)\) \)

4. Decide whether the set \( \{ z : 0 \leq \arg(z) \leq \frac{\pi}{4} \} \) is bounded. Give reasons for your answer.

   \( \text{(Answer: not bounded.)} \)

5. Describe the domain of definition of the function \( f(z) = z/(z + \bar{z}) \).

   \( \text{(Answer: } Re(z) \neq 0. ) \)

6. Find and sketch the images of the hyperbolas

   \[ x^2 - y^2 = -1 \quad \text{and} \quad xy = -2 \]

   under the transformation \( w = z^2 = (x + iy)^2 \).

   \( \text{(Answer: The vertical line } x = -1 \text{ and the horizontal line } y = -4 \text{ respectively.)} \)

7. Show that the function \( f(z) = x^2 + iy^2 \) is differentiable at the origin but analytic nowhere.

8. Find the harmonic conjugate of the function \( u(x, y) = y^3 - 3x^2y \) if it exists. If the answer is yes, determine the analytic function \( f \) whose real part is \( u \).

   \( \text{(Answer: } v(x, y) = -3xy^2 + x^3 + C, f(z) = i(z^3 + C).) \)

9. State whether each of the following statements is true or false. If the statement is true, give a short proof of it. If not, give a counterexample to show that it is false.

   (a) The function \( f(z) = e^z \) is harmonic.

   (b) \( |(2\bar{z} + 5)(\sqrt{2} - i)| = \sqrt{3}|2z + 5| \)
(c) There exists a complex number \( z_0 \) whose fourth roots \( z_1, z_2, z_3, z_4 \) have the property that
\[
\text{arg}(z_1) = \frac{\pi}{4}, \quad \text{arg}(z_2) = \frac{\pi}{2}, \quad \text{arg}(z_3) = \frac{2\pi}{3}, \quad \text{arg}(z_4) = \pi.
\]

(d) The equation \((z^2 + z + 1)e^z = 0\) has exactly two complex roots.

(e) If a rational function \( R \) has a pole at the point \( a \), then the residue of \( R \) at \( a \) must be a nonzero complex number.

(Answer: (a) True (b) True (c) False (d) True (e) False )