1. Does the sequence of functions
   \[ f_n(x) = nx e^{-nx} \]
   converge pointwise on \([0, \infty)\)? Is the convergence uniform on this interval? If yes, give reasons. If not, determine the intervals (if any) on which the convergence is uniform.

2. Let \( \{f_n : n \geq 1\} \) and \( \{g_n : n \geq 1\} \) be real-valued functions on a set \( X \), and suppose that both sequences converge uniformly on \( X \). Show that the sequence \( \{f_n + g_n : n \geq 1\} \) converges uniformly on \( X \). Give an example showing that \( \{f_ng_n : n \geq 1\} \) need not converge uniformly on \( X \).

3. Fix \( a, b \in \mathbb{R}, a < b \). Let \( f_n : [a, b] \to \mathbb{R} \) satisfy \( |f_n(x)| \leq 1 \) for all \( x \) and \( n \). Show that there is a subsequence \( \{f_{n_k}\} \) such that \( \lim_{k \to \infty} f_{n_k}(x) \) exists for each rational \( x \in [a, b] \).