Quiz 1 - Math 105, Section 204

Name: **SID:**

Date: Jan 11, 2012 **Time: 20 mins**

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1. Does the equation $x + y = 1$ represent a line or a plane in (x, y, z)-space? Sketch the graph of this equation.

   (5 points)

   Solution: We have 1 linear equation, so it must be a plane.

   (2 points)

   In the (x, y)-plane, the equation $x + y = 1$ is a line passing through $(1, 0, 0)$ and $(0, 1, 0)$.

   (2 points)

   Since z is not in the equation, it is a free variable, so we get a plane parallel to the z-axis containing the line through $(1, 0, 0)$ and $(0, 1, 0)$.

   (1 point)

2. Find a unit vector that is parallel to the normal vector of the plane $2x - y + z = 10$.

   (5 points)

   Solution: The normal vector of the plane is $\langle 2, -1, 1 \rangle$.

   (1 point)

   To get a vector parallel to it, we need to scale it by a constant factor.

   (1 point)

   To get a unit vector, we need the length to be 1. The length of $\langle x, y, z \rangle$ is $\sqrt{x^2 + y^2 + z^2}$, so the length of $\langle 2, -1, 1 \rangle$ is $\sqrt{6}$.

   (2 point)

   Therefore a unit vector parallel to the normal vector is $\langle 2/\sqrt{6}, -1/\sqrt{6}, 1/\sqrt{6} \rangle$. (Another one is $\langle -2/\sqrt{6}, 1/\sqrt{6}, -1/\sqrt{6} \rangle$.)

   (1 point)

3. Determine whether the two vectors $\langle 3, -1, 1\rangle$ and $\langle 2, 5, -1\rangle$ are parallel, perpendicular or neither.

(5 points)

Solution: For two vectors to be parallel, one must be a constant multiple of another. However, this is not the case since $3 \cdot 2/3 = 2$, and $-1 \cdot 2/3 = -2/3 \neq 5$. Therefore the vectors are not parallel.

(2 points)

For two vectors to be perpendicular, their dot product must be 0. The dot product of the two given vectors is

$$3 \cdot 2 + (-1) \cdot 5 + 1 \cdot (-1) = 6 - 5 - 1 = 0.$$

Therefore the vectors are perpendicular.

(3 points)