What is a sequence?

Definition

A sequence $\{a_n\}$ is an ordered list of numbers of the form

$$\{a_1, a_2, a_3, \cdots, a_n, \cdots\}.$$

Remarks and examples:

1. There has to be a well-defined rule or pattern for generating the numbers of a sequence. For example, the sequence

$$\left\{\frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \frac{1}{16}, \cdots\right\}$$

is generated using the **explicit formula**

$$a_n = \frac{1}{4n}, \qquad n \geq 1.$$

Examples of sequences (ctd)

2. Even if an explicit formula is not available, for example

$$\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \cdots, \}$$

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a sequence in equally well-defined via a **recurrence relation** that describes the n^{th} term of the sequence in terms of its predecessors. In the above example of the famous **Fibonacci sequence**, the recurrence formula is

$$a_{n+2} = a_n + a_{n+1}, \qquad a_1 = 0, \quad a_1 = 1.$$

Limit of a sequence

Definition

If the terms of a sequence $\{a_n\}$ approach a unique numbe L as n increases, then we say

$$\lim_{n\to\infty} a_n = L$$

exists, and that the sequence $\{a_n\}$ **converges** to L. If the terms of the sequence approach $+\infty$ or $-\infty$ or do not approach a single number as n increases, the sequence has no limit, and the sequence is said to **diverge**.

Techniques for computing limits

1. Converting limits of sequences to limits of functions and applying L'Hôpital's rule (when applicable)
   - An example
   - Discussion of L'Hôpital's rule

2. New limits from old, using algebraic rules for sums, differences, products and quotients of limits
   - An example
   - Review of limit algebra

3. Geometric sequence $\{r^n : n \geq 1\}$
   - Discussion of the cases $|r| < 1$, $r = \pm 1$ and $|r| > 1$.

4. Squeeze theorem
   - Statement of the result
   - An example

Does this sequence have a limit?

Find the limit of the sequence

$$a_n = \frac{\sin^3\left(\frac{(2n+1)\pi}{2}\right)}{(n+1)\sin\left(\frac{1}{n}\right)}$$

if it exists.

A. 1

B. 0

C. -1

D. does not exist