# A quick review

A production function $P$ computes the units of goods produced, given $L$ units of labor and $K$ units of capital.

$$P(K, L) = KL + K^2 + 3L^2.$$

A firm with this production function currently employs 25 units of capital and 10 units of labor. By approximately how many units would the production of the company change if it increased its labor units to 12 but reduced its capital to 23?

A. Production stays the same

B. Production increases by 50 units

C. Production decreases by 50 units

D. Production increases by 230 units

# Relative maxima and minina

## Definition

Given a function $f(x,y)$ of two variables,

- We say that $f$ has a **local maximum** at the point $(a,b)$ if

$$f(x,y) \leq f(a,b)$$

  for all $(x,y)$ close enough to $(a,b)$.

- We say that $f$ has a **local minimum** at the point $(a,b)$ if

$$f(x,y) \geq f(a,b)$$

  for all $(x,y)$ close enough to $(a,b)$.

# Relative maxima and minina

## Definition

Given a function $f(x, y)$ of two variables,

- We say that $f$ has a **local maximum** at the point $(a, b)$ if

$$f(x, y) \leq f(a, b)$$

  for all $(x, y)$ close enough to $(a, b)$.

- We say that $f$ has a **local minimum** at the point $(a, b)$ if

$$f(x, y) \geq f(a, b)$$

  for all $(x, y)$ close enough to $(a, b)$.

**Today's goal:** Given a function $f$, identify its local maxima and minima.

# The first derivative test

## Description of the test

The first step in finding local max or min of a function $f(x, y)$ is to find points $(a, b)$ that satisfy the two equations

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$

Any such point $(a, b)$ is called a **critical point of $f$**.

# The first derivative test

## Description of the test

The first step in finding local max or min of a function $f(x, y)$ is to find points $(a, b)$ that satisfy the two equations

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$

Any such point $(a, b)$ is called a **critical point of** $f$.

**Note:** Any local max or min of $f$ has to be a critical point, but every critical point need not be a local max or min.

# Finding critical points : an example

Find all critical points of the following function

$$f(x, y) = \frac{1}{x} + \frac{1}{y} + xy.$$

A. $(0, 0), (1, 1)$

B. $(1, 1)$

C. $(0, 0), (1, 1), (1, -1), (-1, 1), (-1, -1)$

D. There is no critical point

E. $(0, 0)$

# The previous example (ctd)

Is the critical point $(1, 1)$ a local max, a local min or neither?

## The second derivative test

If $(a, b)$ is a critical point of $f$, calculate $D(a, b)$, where

$$D = f_{xx} f_{yy} - f_{xy}^2.$$

1. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f$ has a local maximum value at $(a, b)$.
2. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f$ has a local minimum value at $(a, b)$.
3. If $D(a, b) < 0$, then $f$ has a saddle point at $(a, b)$.
4. If $D(a, b) = 0$, then the test is inconclusive.

# Classifying critical points : an example

In the example

$$f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$$

determine whether the critical point $(1,1)$ is

A. a local minimum

B. a local maximum

C. a saddle point

D. neither of the above

# An application

A company manufactures two products $A$ and $B$ that sell for \$10 and \$9 per unit respectively. The cost of producing $x$ units of $A$ and $y$ units of $B$ is

$$400 + 2x + 3y + 0.01(3x^2 + xy + 3y^2).$$

Find the values of $x$ and $y$ that maximize the company's profits.

A. $(100, 80)$

B. $(120, 90)$

C. $(120, 80)$

D. $(80, 120)$