Closed book examination  

Time: 50 minutes

Last Name ________________ First ___________  SID _____________

Instructor name: Keqin Liu

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in “calculator-ready” form. Simplification of the final answer is worth at most one point.

2. Show all your work. A correct answer without accompanying work will get no credit.

3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBC card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  - Speaking or communicating with other candidates.
  - Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

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<tr>
<th>Q</th>
<th>Points</th>
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<td>4 (extra credit)</td>
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1. (a) Find the derivative of the function

\[ f(x) = \int_{x}^{x^2} \cos(t^2) \, dt \]

at the point \( x = 0 \).  

(10 points)
(b) Use Simpson’s rule to approximate

\[ \int_{1}^{2} \frac{dx}{x} \]

with \( n = 4 \) subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)
(c) Find the definite integral

\[ \int_{0}^{\pi} \sec^2 x \, dx. \]

(10 points)
(d) A student randomly guesses at each answer in a true/false quiz consisting of 3 questions. Let $X$ be the random variable representing the number of correct answers. Find the probability density function of $X$. 

(10 points)
(e) Find the indefinite integral

\[ \int \sin^3(x) \cos^{10}(x) \, dx. \]

(10 points)
(f) Solve the initial value problem

\[ e^{-t}y' = \frac{t}{y}, \quad y(0) = -5. \]

(10 points)
2. Evaluate the definite integral:

\[ \int_0^{\ln(\sqrt{3})} \frac{e^{2t}}{(1 + e^{4t})^{\frac{3}{2}}} \, dt. \]

(20 points)
3. The time to failure of a transistor (in years) is a continuous random variable whose cumulative distribution function is given by

\[ F(x) = \begin{cases} 
1 - e^{-mx} & \text{if } x \geq 0, \\
0 & \text{otherwise,} 
\end{cases} \]

where \( m \) is an unknown constant.

(3 + 7 + 10 = 20 points)

(a) Find the probability density function of \( X \).

(b) If the expected time to failure of a transistor is 10, find \( m \).
(c) What is the probability that a transistor will last for at least 15 years?
4. (Extra credit) The monthly average price of silver has been growing at a rate proportional to the square root of the price since November 2011. The average price in November 2011 of one gram of silver was $16 and the average price of the same in February 2012 was $25. Write down the initial value problem of the monthly value of silver as a function of time. **Do not solve this problem!**

(5 points)
Formula Sheet

You may refer to these formulae if necessary.

**Trigonometric formulae:**

\[
\cos^2 x = \frac{1 + \cos(2x)}{2},
\]
\[
\sin^2 x = \frac{1 - \cos(2x)}{2}.
\]

**Simpson’s rule:**

\[
S_n = \frac{\Delta x}{3} \left( f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n) \right).
\]
\[
E_s = \frac{K(b - a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a, b].
\]

**Indefinite Integrals:**

\[
\int \sec x \, dx = \ln | \sec x + \tan x | + C.
\]

**Probability:**

\[
\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) \, dx.
\]
\[
\text{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x) \, dx.
\]