MATH 105 Practice Problem Set 2 Questions

1. Parts (a)–(d) are TRUE or FALSE, plus explanation. Give a full-word answer TRUE or FALSE. If the statement is true, explain why, using concepts and results from class to justify your answer. If the statement is false, give a counterexample.

(a) [5 marks] The level curves of the plane $ax + by + cz = d$, where $a, b, c, d \neq 0$, are parallel lines in the $xy$-plane.

(b) [5 marks] The domain of the function $g(x, y) = \ln ((x + 1)^2 + (y - 2)^2 - 1)$ consists of all points $(x, y)$ lying strictly in the interior of a circle centered at $(-1, 2)$ of radius 1.

(c) [5 marks] There exists a function $f(x, y)$ defined on $\mathbb{R}^2$ such that $f_x(x, y) = \cos (3y)$ and $f_y(x, y) = x^4$.

(d) [5 marks] If $f(x, y)$ is a function such that $f_x(x, y) = 2x + y$ and $f_y(x, y) = x + 1$,
then $f$ is differentiable at every point in $\mathbb{R}^2$. 
2. Consider the surface \( zx^2 = z^2 - y^2 \).

(a) \( 10 \) marks Find the equations and sketch the level curves for \( z = -1, 0, 1 \) on the same set of axes.

(b) \( 5 \) marks Find all values of \( z \) which correspond to a level curve containing the point \( (x, y) = (2, 0) \).
3. 10 marks  Show that \( u(x, t) = t^{-\frac{1}{2}} e^{-\frac{x^2}{4t}} \) is a solution to the heat equation

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}
\]
4. Consider the Body Mass Index function being calculated by $b(w, h) = \frac{w}{h^2}$, where $w$ is the weight in kilograms and $h$ is the height in meters.

(a) 5 marks Compute $b_w$ and $b_h$.

(b) 5 marks For a fixed weight, as the height increases, how does the Body Mass Index change? Explain using the answers in part a.
5. Let \( f(x, y) = y^3 \sin(4x) \).

(a) 5 marks Explain in your own words what it means for the function \( f(x, y) \) to be differentiable at a point \((a, b)\).

(b) 5 marks Show that \( f \) is differentiable at every point in \( \mathbb{R}^2 \).
6. 15 marks Find the maximum and minimum values of the function $f(x, y) = ye^x - e^y$
in the area bounded by the triangle whose vertices are (4, 1), (1, 1) and (4, 4).
7. [10 marks] Find the maximum and minimum of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$. 
8. Let \( f(x) = 2x + 1 \).

(a) 5 marks Write down the left Riemann sum for \( \int_1^5 f(x) \, dx \).

(b) 10 marks Compute the limit as \( n \to \infty \) of the Riemann-sum expression found in part (a) and thus evaluate \( \int_1^5 f(x) \, dx \).